Approximate Homomorphic Encryption

MOTIVATION, CONSTRUCTION, APPLICATIONS

YONGSOO SONG

UCSD

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Reduction of "Gap" between Real and Encrypted computations.

Datatypes and operations

- Boolean Circuit (Bit Operation)
- Integer Operations
- Modular Arithmetic
- Approximate Arithmetic (Fixed/Floating-point Operation)
- Logical Operations (If & Else statement)

Homomorphic Encryption Schemes

Scheme	Plaintext Slot	Good	Bad	Library
"Word Encryption" Brakerski-Gentry-Vaikuntanathan'12 Gentry-Halevi-Smart'12a,b,c Brakerski'12, Fan-Vercauteren'12 Halevi-Shoup'13,14,15	$GF(p^d)$ (Z_p)	Polylog overhead (Amortized time & Expansion rate)	Bootstrapping	HElib SEAL …
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Application Researches of HE (2017~2018)

“Homomorphic Encryption” in ePrint and IEEE Xplore

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◦ Neural Network:	2	(2018/073, 2017/1114)
◦ Genomic Data:	7	(2017/955,770,294,228. EUSIPCO, SMARTCOMP, IEEE Journal)
◦ Health Data:	2	(IBM Journal, IEEE Journal)
◦ Biometric Data:	2	(IEEE Access, IEEE Conference)
◦ Energy Management:	3	(2017/1212. IEEE Big Data, IET Journal)
◦ Big Data:	1	(ICBDA)
◦ Advertising:	1	(WIFS)
◦ Internet of Things:	1	(IWCMC)
◦ Election:	1	(2017/166)

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1,234 * 689 * 2,194 * 917 * 3,323 * 4,154 * 489 * 3,772 = 4,355,296,408,921,213,975,719,328 > 2^{85}.

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- Express a real number as a (small) polynomial. e.g. (1.234) → ($1 + 2X^{-1} + 3X^{-2} + 4X^{-3}$)

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- Exponential growth of Degree
- Trade-off between Precision & Number of slots
- No Bootstrapping

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- 0.06 sec for (2-to-1) gate.
- 10 sec for (6-to-6) circuit.

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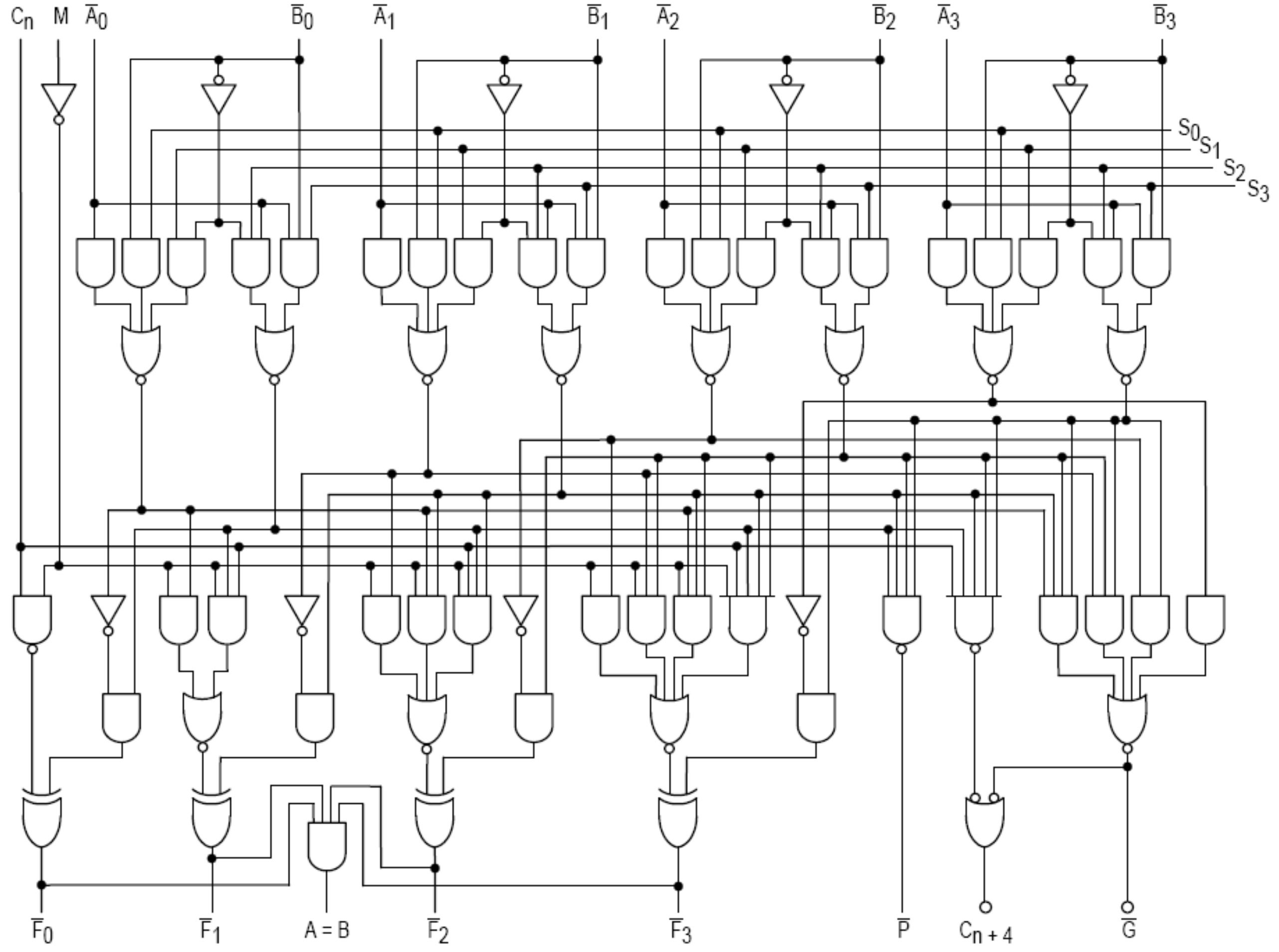
75 Gates for an operation on two four-bit strings.

How many gates for
16-bit / 32-bit precision multiplication?



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"Approximate Encryption" Cheon-Kim-Kim-Song'17 Cheon-Han-Kim-Kim-Song'18	Complex (Real) Numbers	Fixed-point Arithmetic. Polylog overhead.		HEAAN (慧 眼)

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Motivation: Imitate the approximate arithmetic on computer system.

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Approximate HE: (1.234) ⇒ (scale by $p=10^4$) ⇒ 12,340.

⇒ (Encrypt) ⇒ $[\langle ct, sk\rangle]_q$ = 12,342 ≈ $1.234 * 10^4$.

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Idea 2. Approximate Rounding is easy!

$\langle ct, sk \rangle = m \pmod q$

$ct \mapsto ct' = \lceil p^{-1} * ct \rfloor$

$\langle ct', sk \rangle \pmod{p^{-1} q} \approx p^{-1} m$

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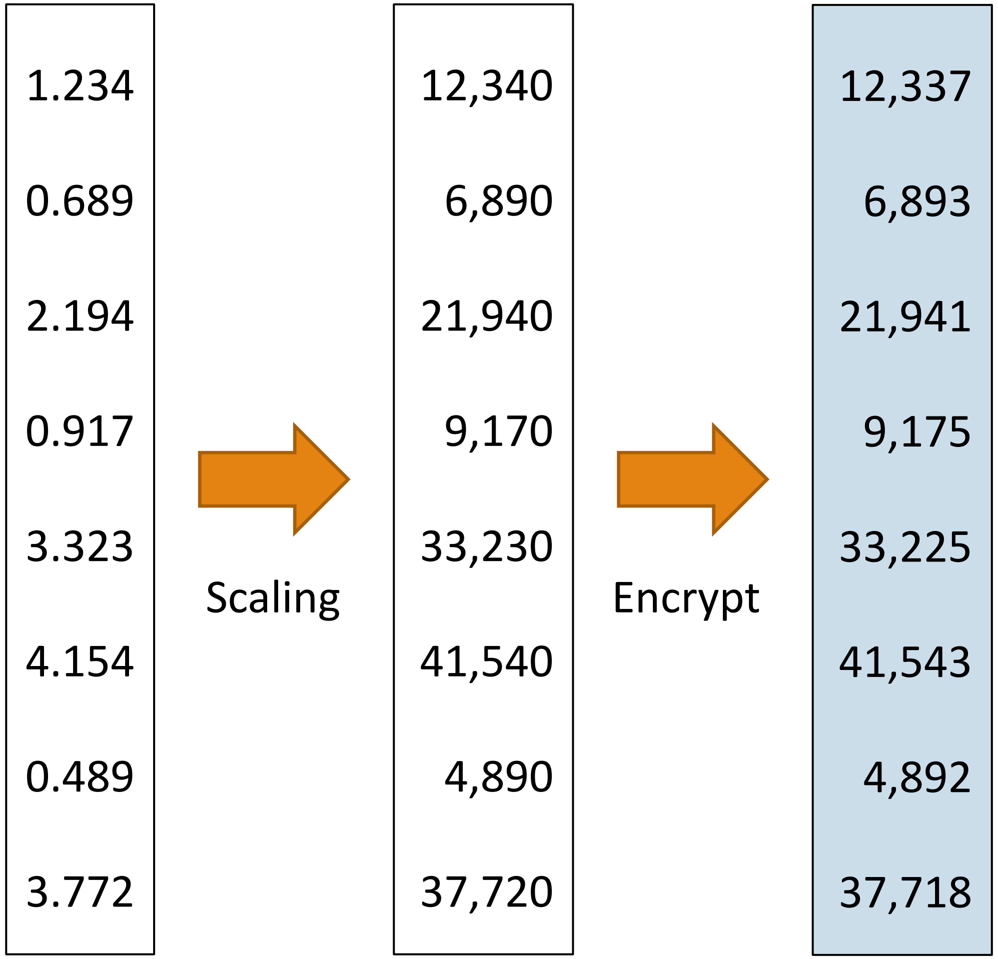

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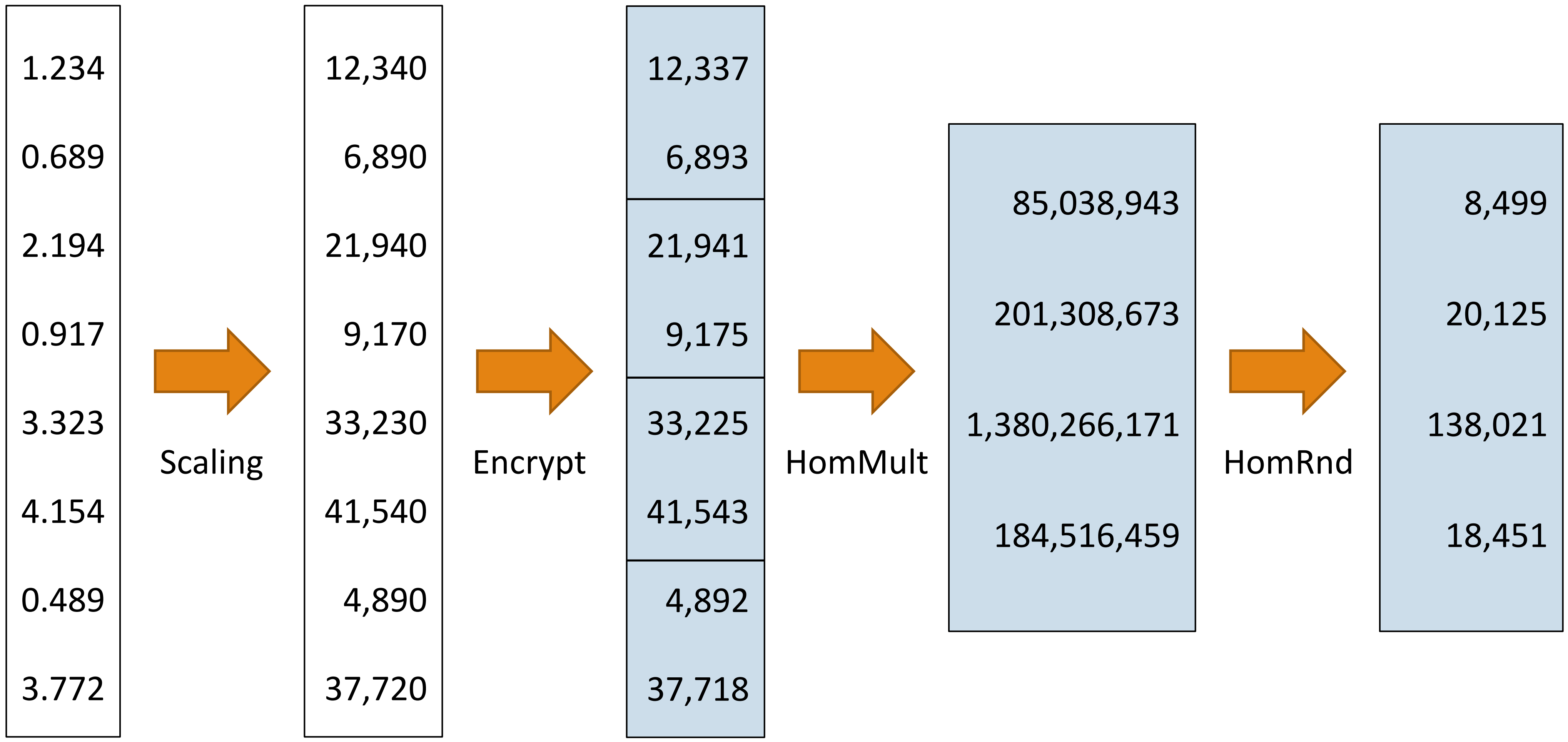

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8,499 20,125
138,021 18,451

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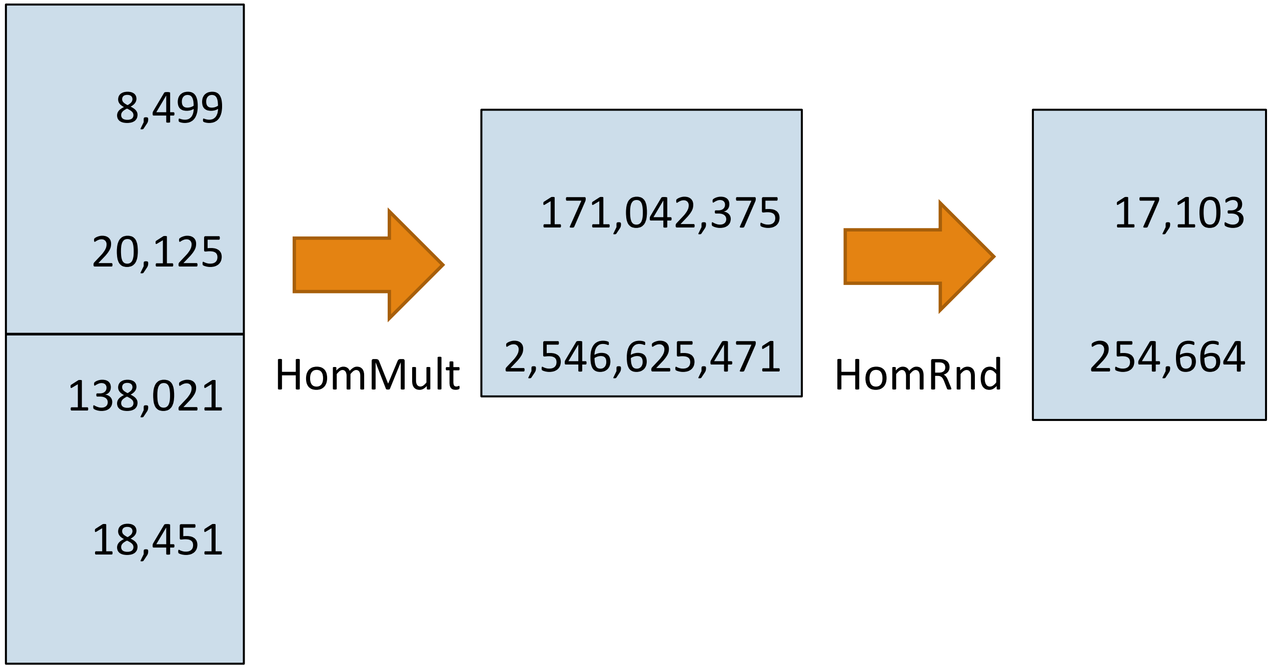

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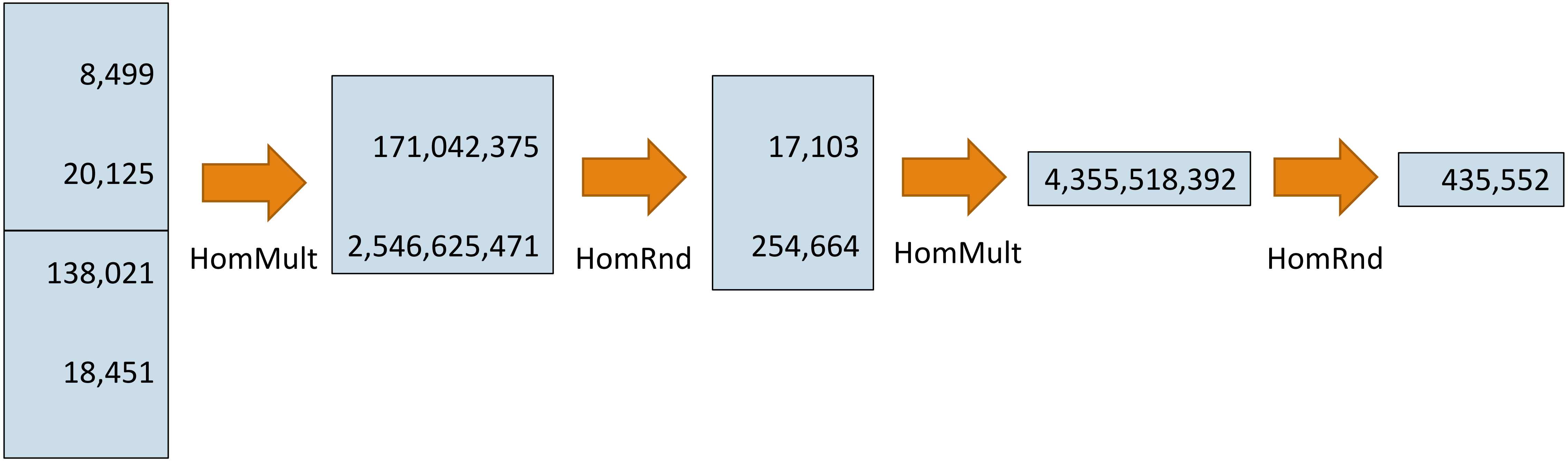

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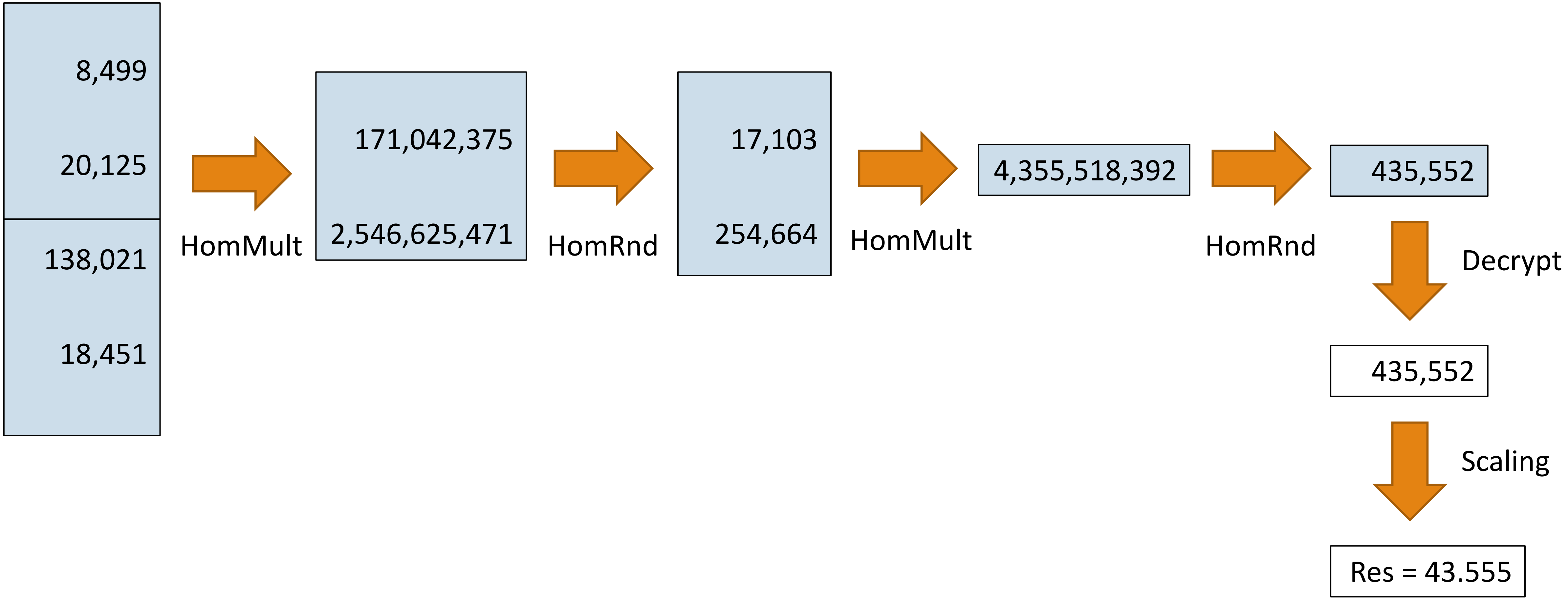

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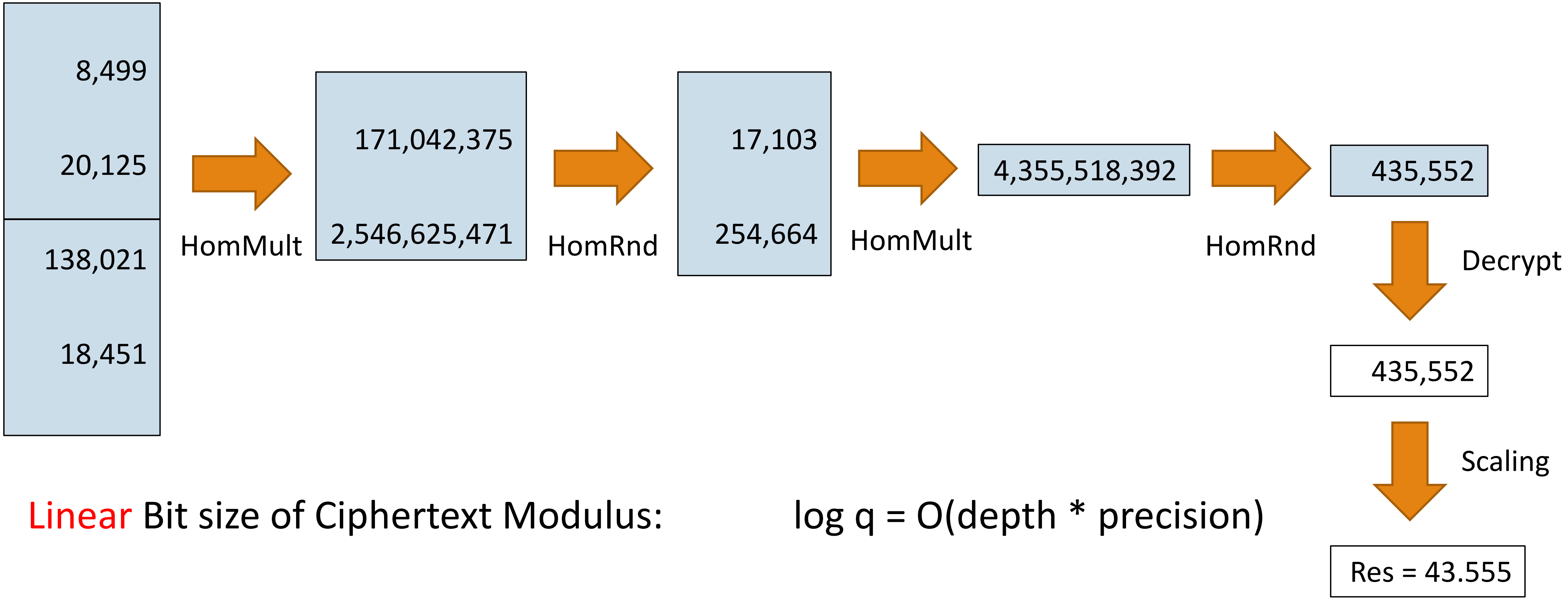

Linear Bit size of Ciphertext Modulus: log q = O(depth * precision)

Functionality of Approximate HE

Packing Technique

- $R = Z[X] / (\Phi_m(X))$.
- $\Phi(X) = \prod_i (X - \zeta_i)$ where ζ_i's are m-th roots of unity.
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Evaluation of Analytic Functions

- $\exp (z)$,
- z^{-1}

Bootstrapping for the Approximate HE (EC'18)

Decryption circuit

- M = <ct, sk> (mod q).
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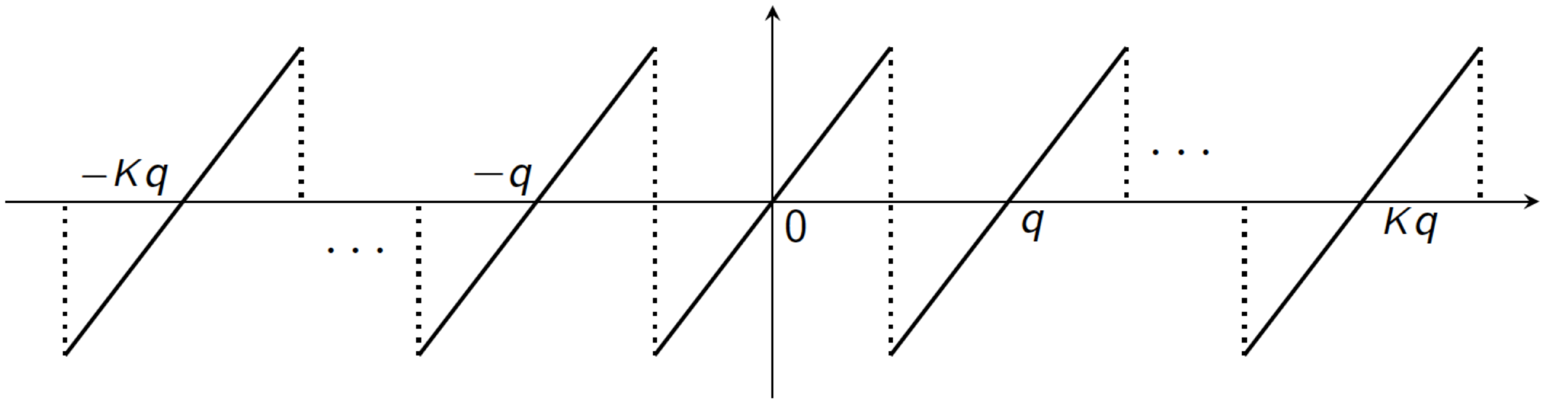

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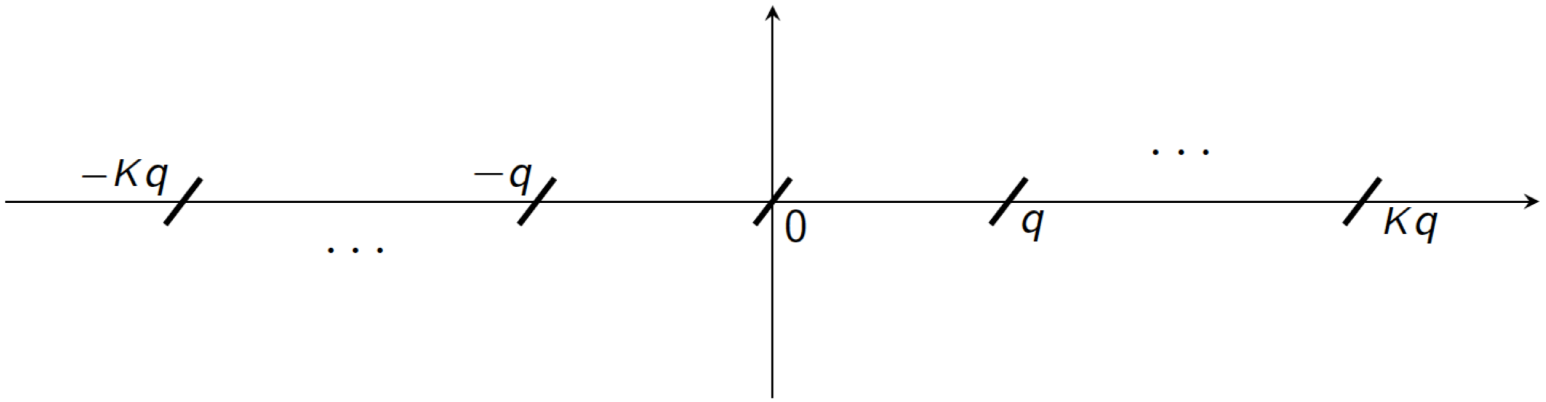

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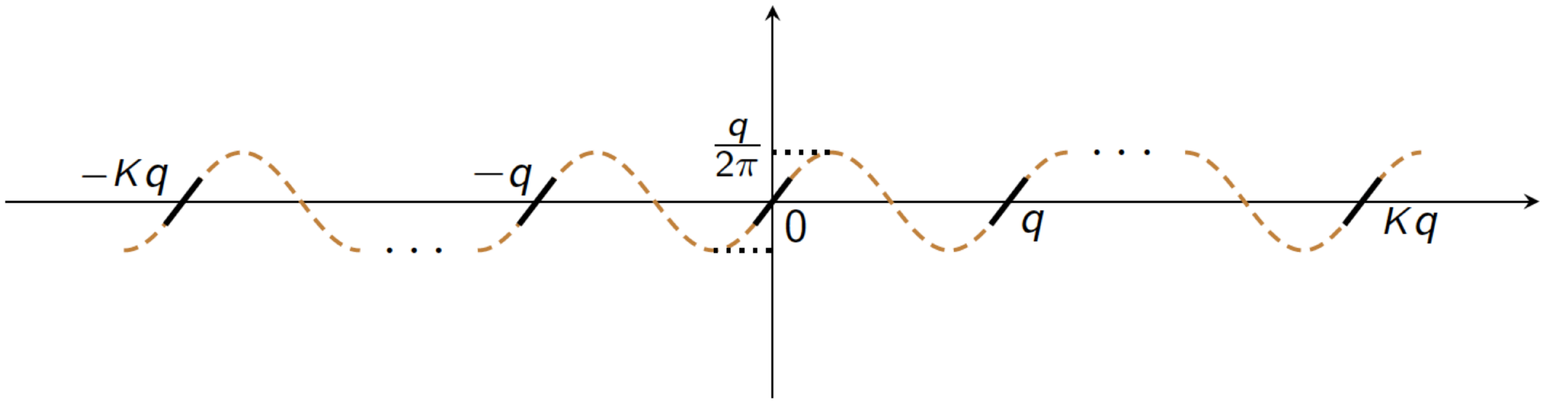

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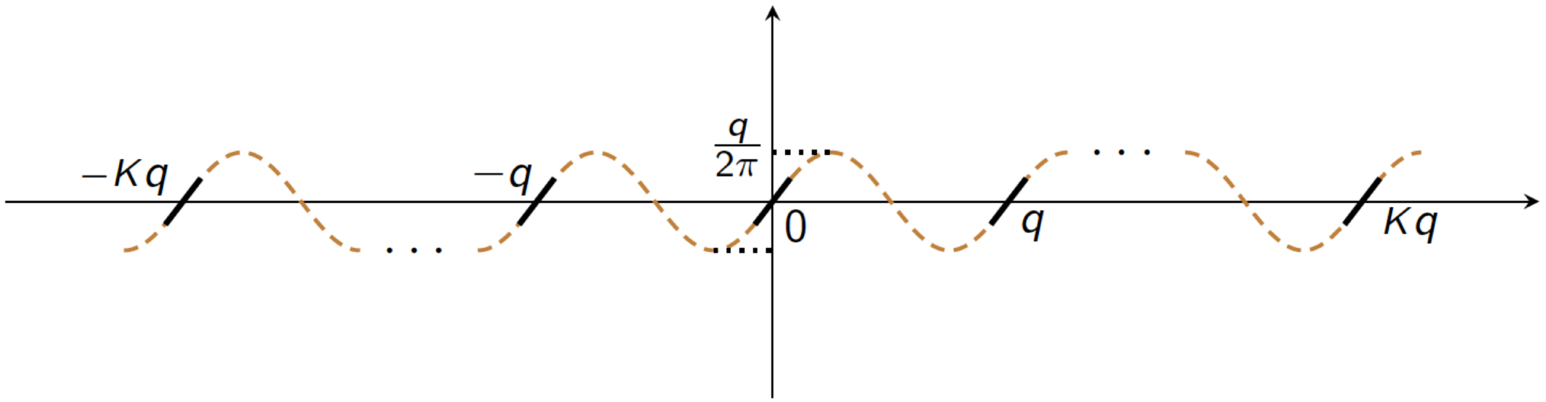

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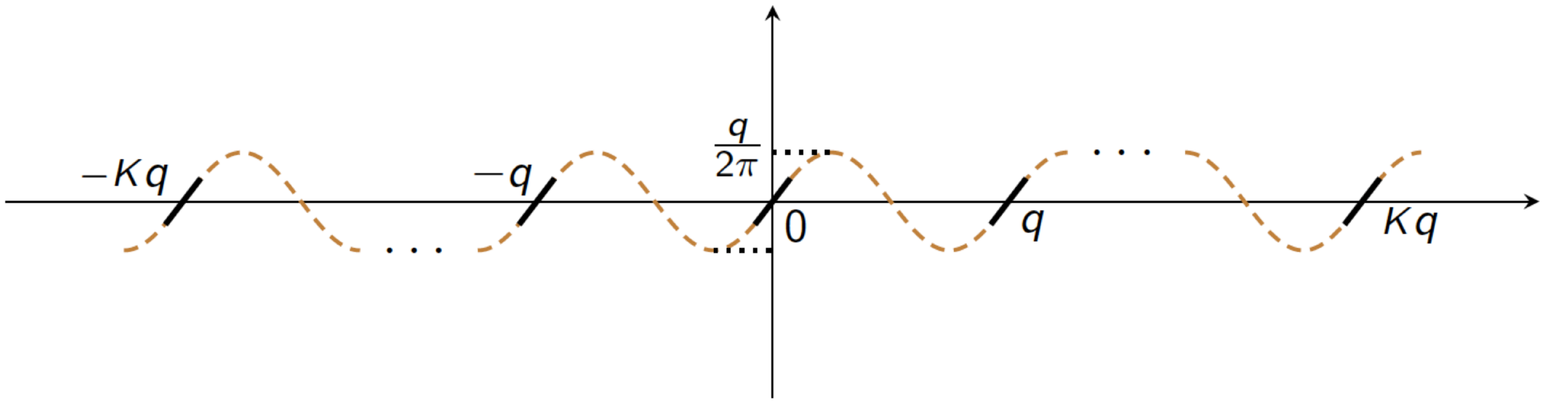

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- From $[-2K\pi/2^r, 2K\pi/2^r]$ to $[-2K\pi, 2K\pi]$.
- Linear Complexity for Modulus Reduction Operation!
- <ct',sk> (mod Q) ≈ M

Following Work

Privacy-preserving Training of Logistic Regression Model

- Kim-Song-Wang-Xia-Jiang, JMIR Med Inform'18
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- Implementation without CRT composition or big-integer library.
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Open problems??

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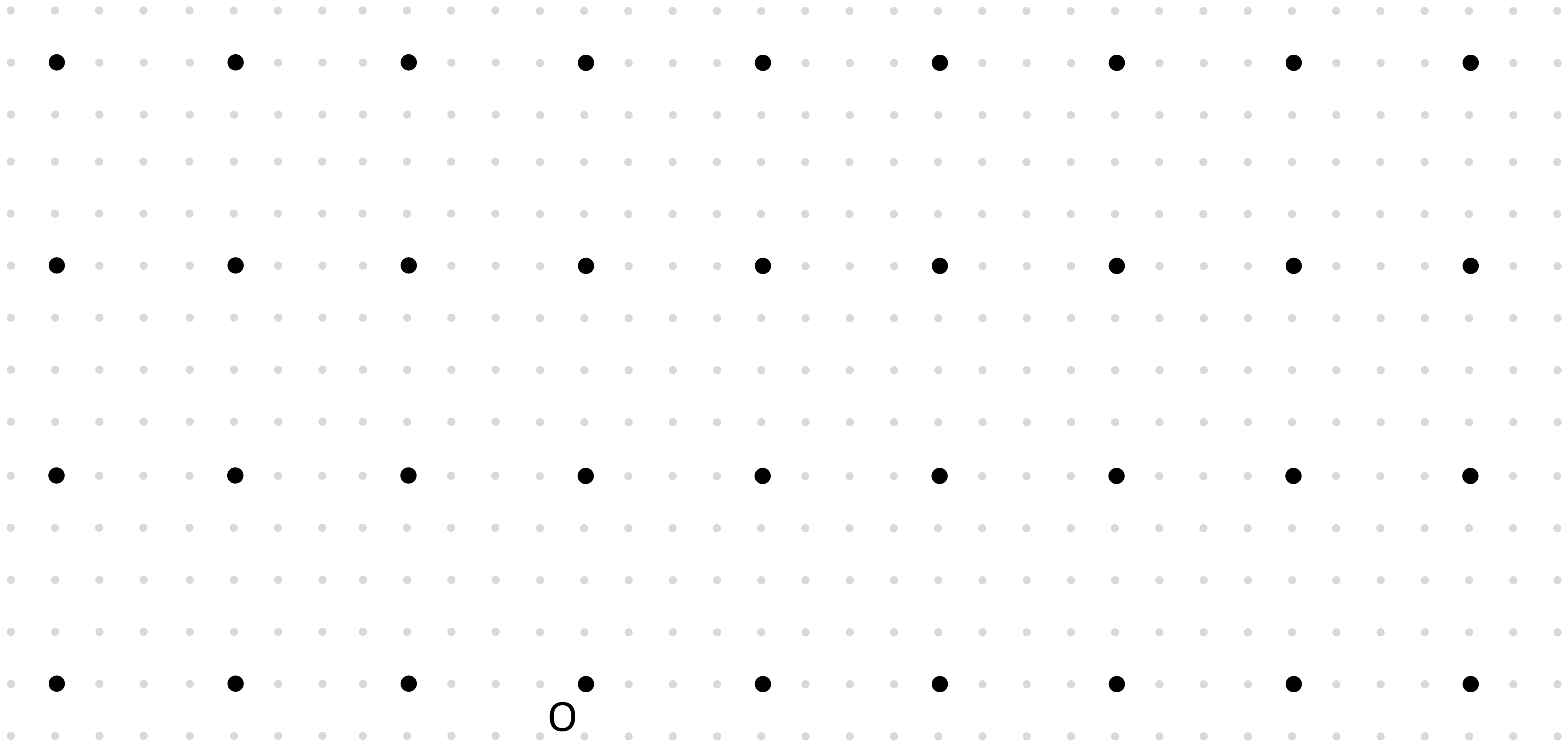

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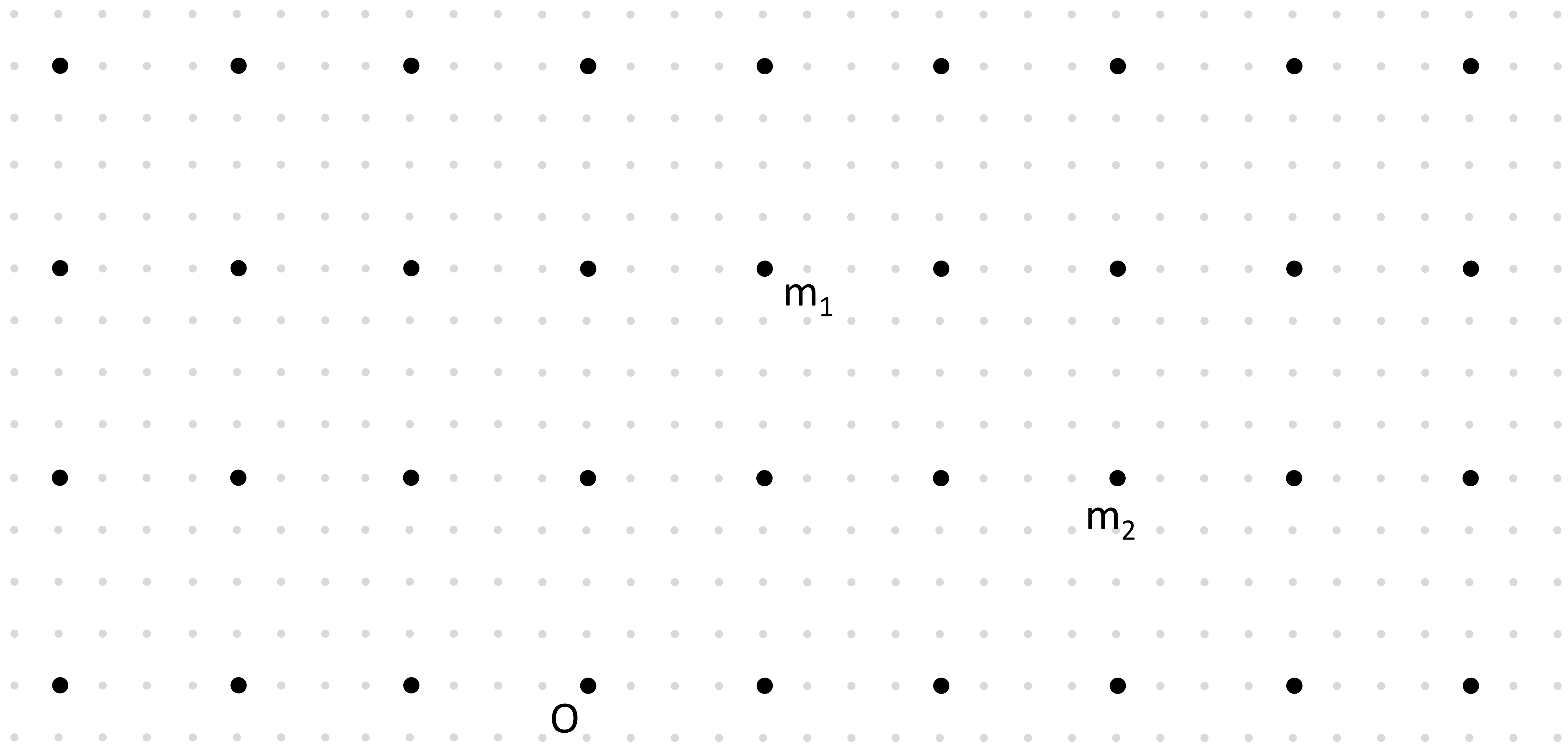
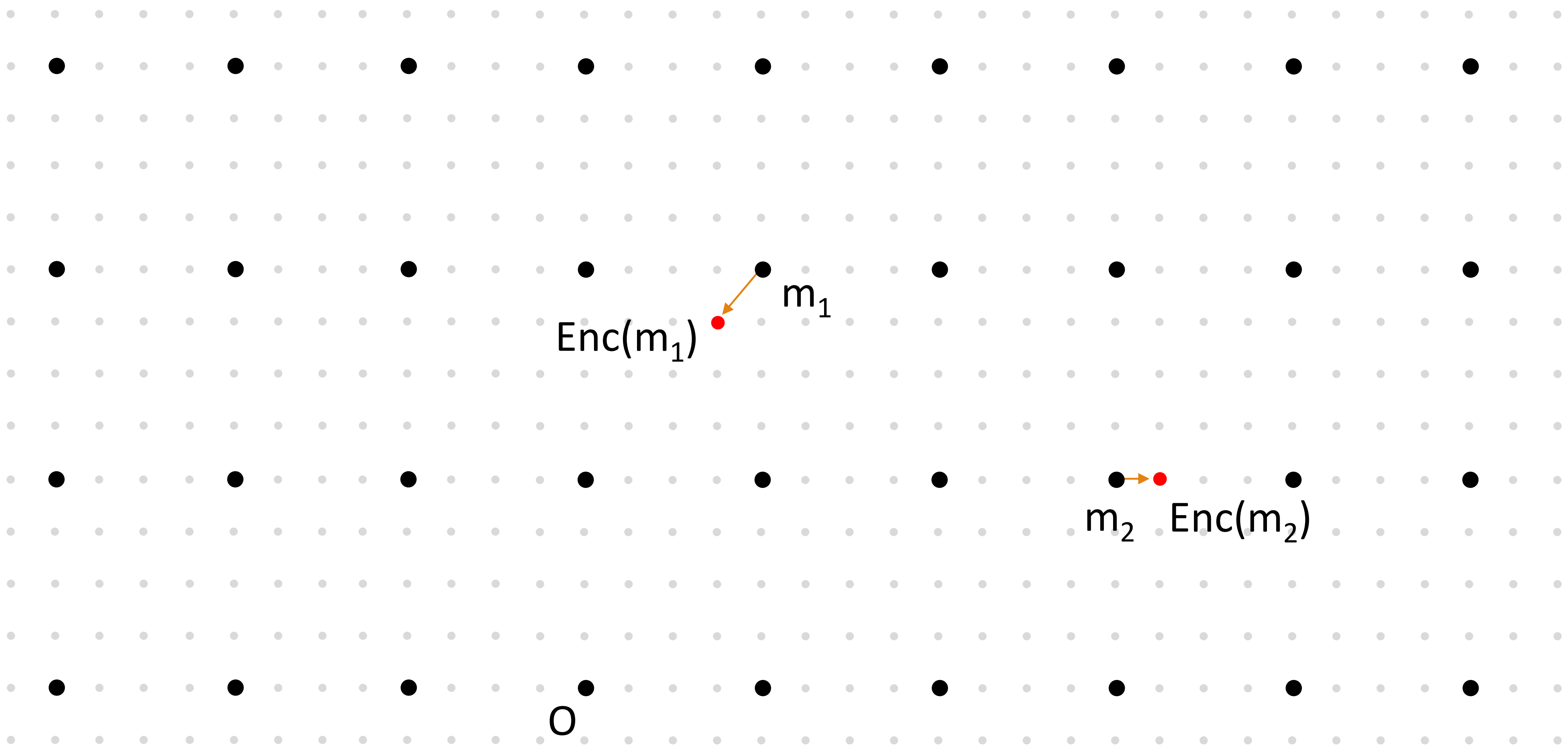


Homomorphic Encryption Framework (Encryption)



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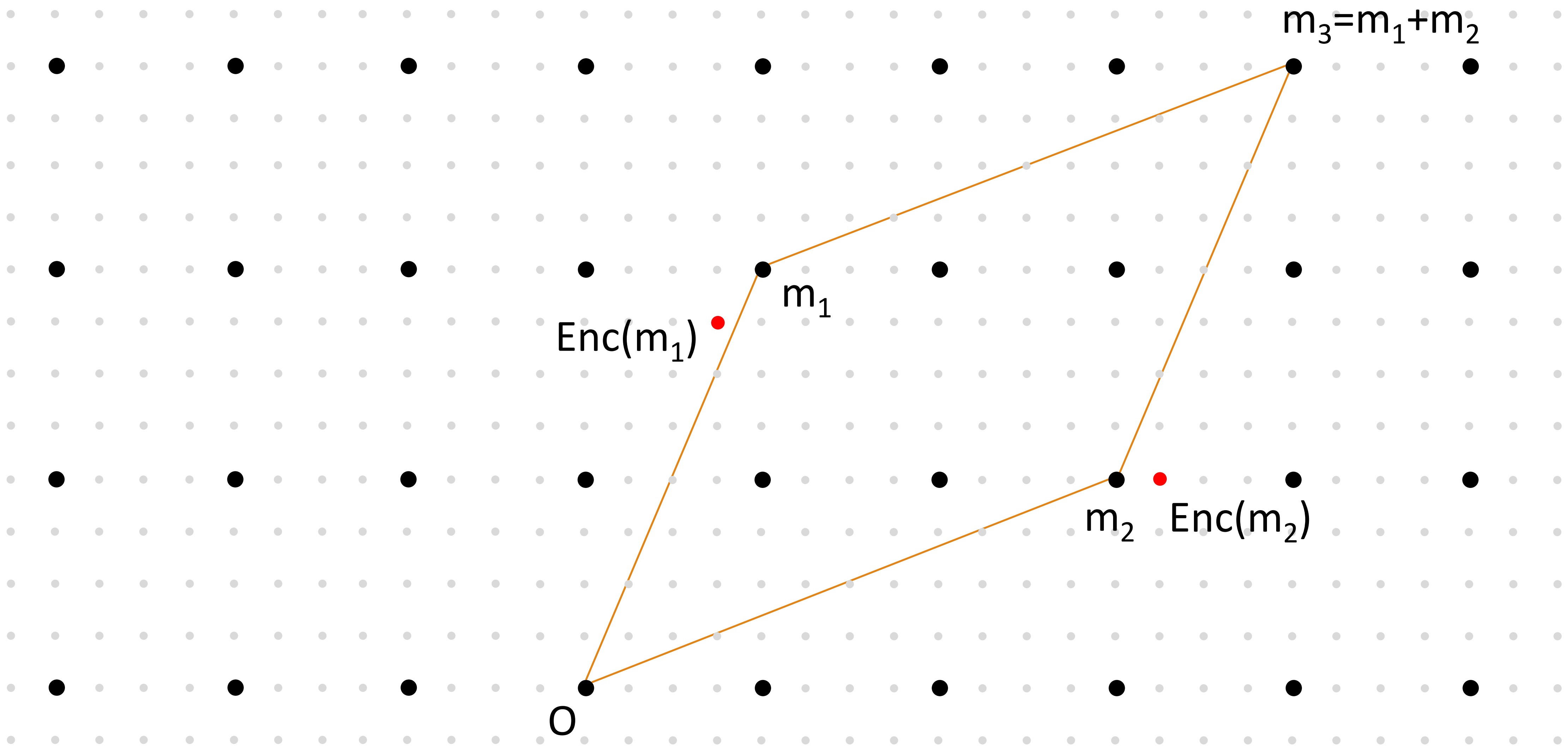

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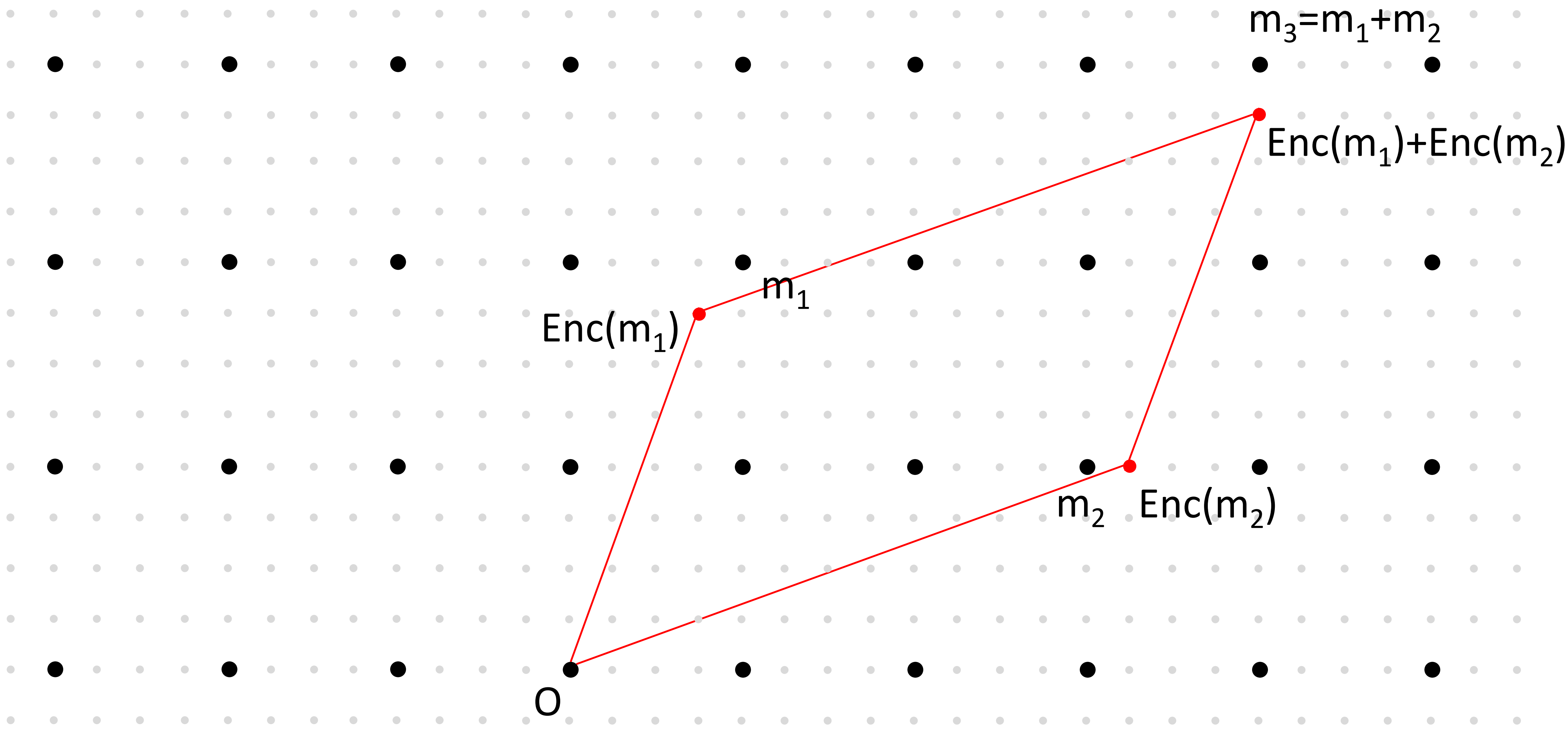

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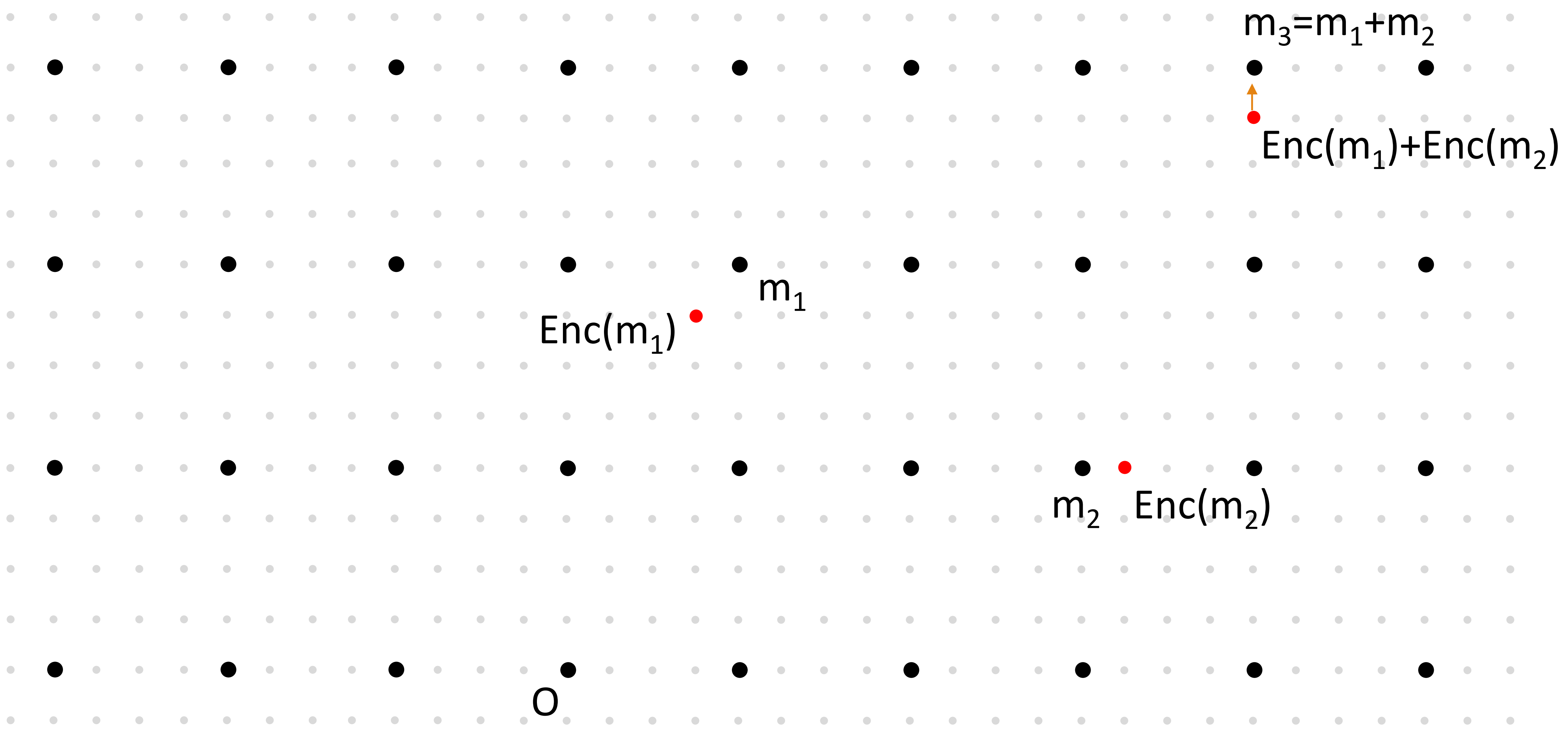

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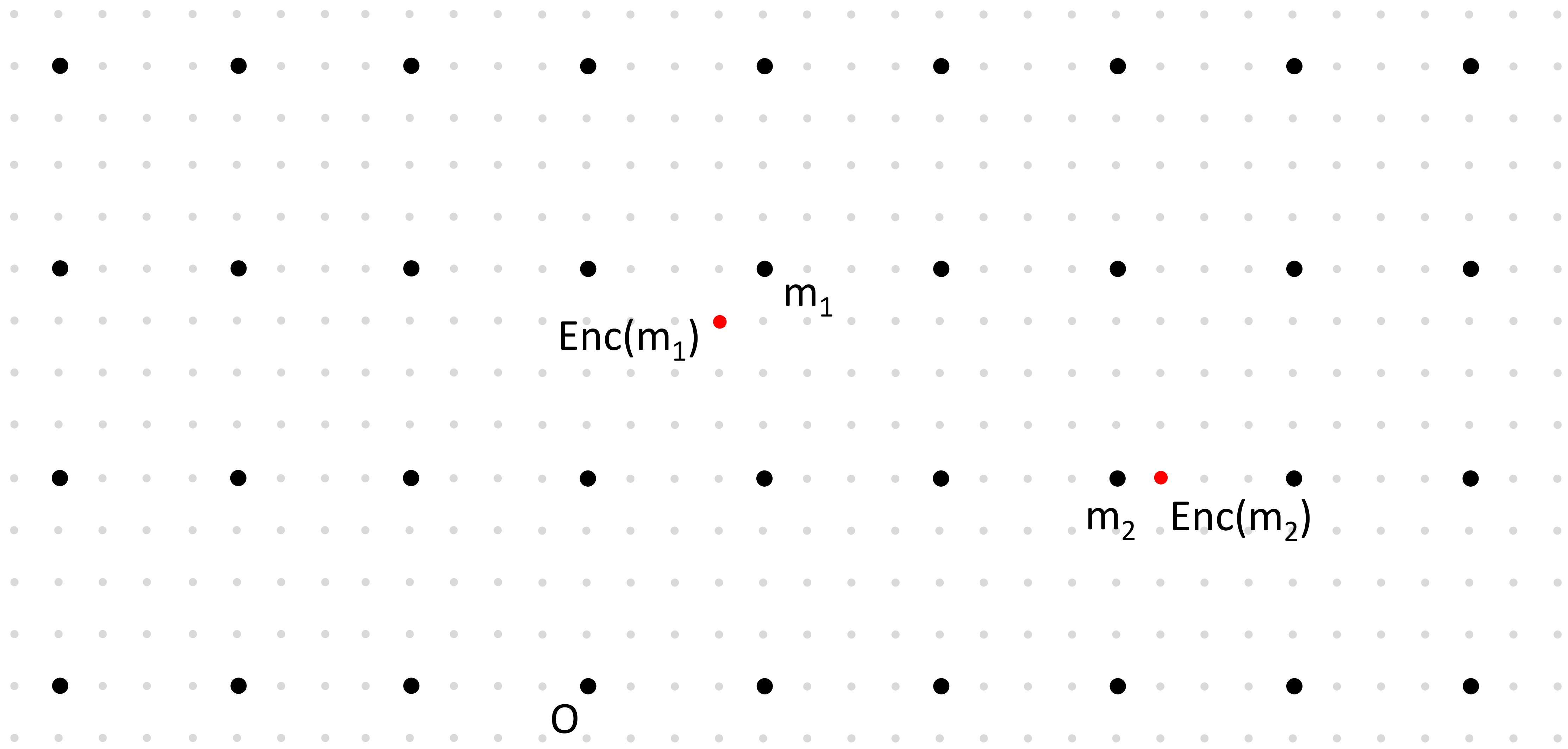

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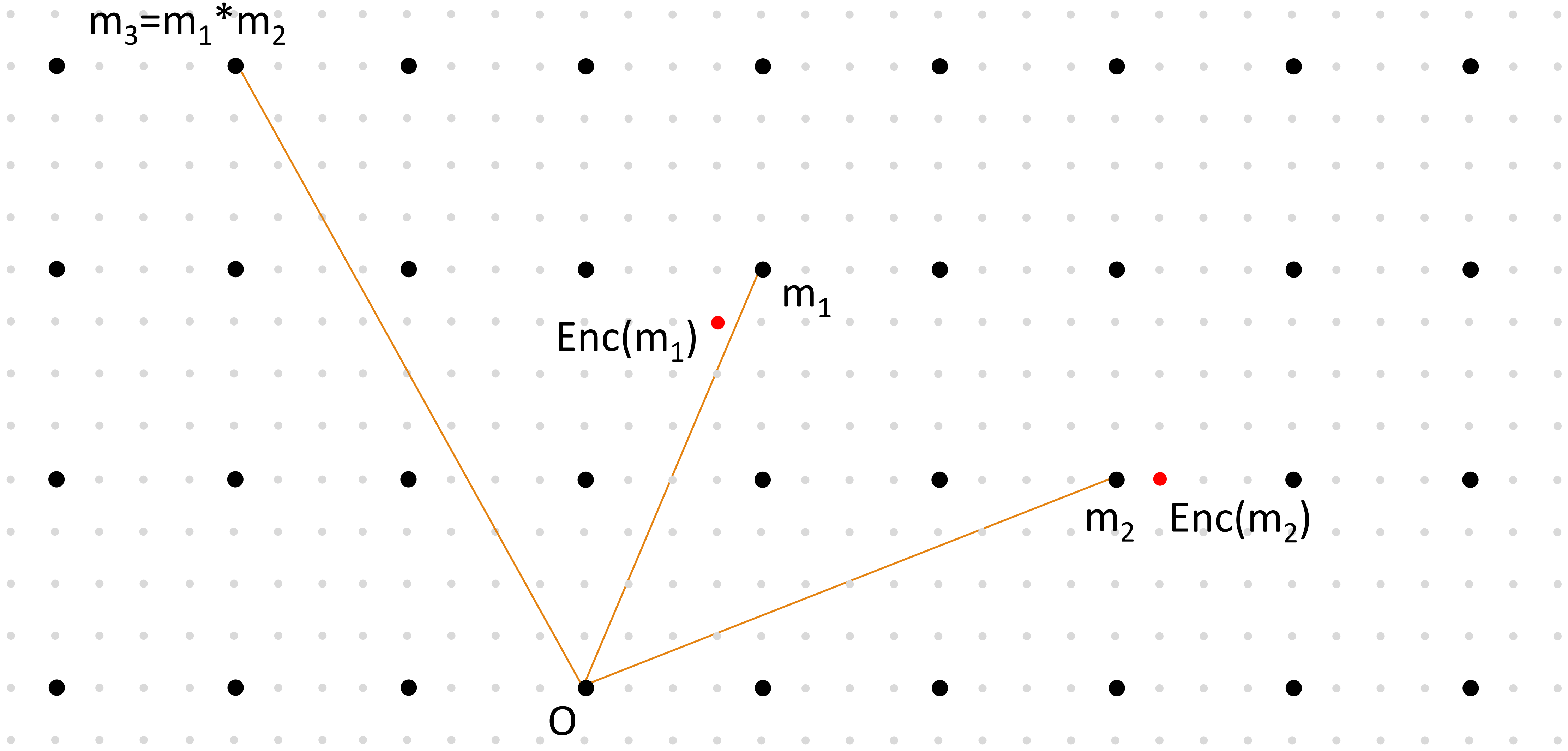

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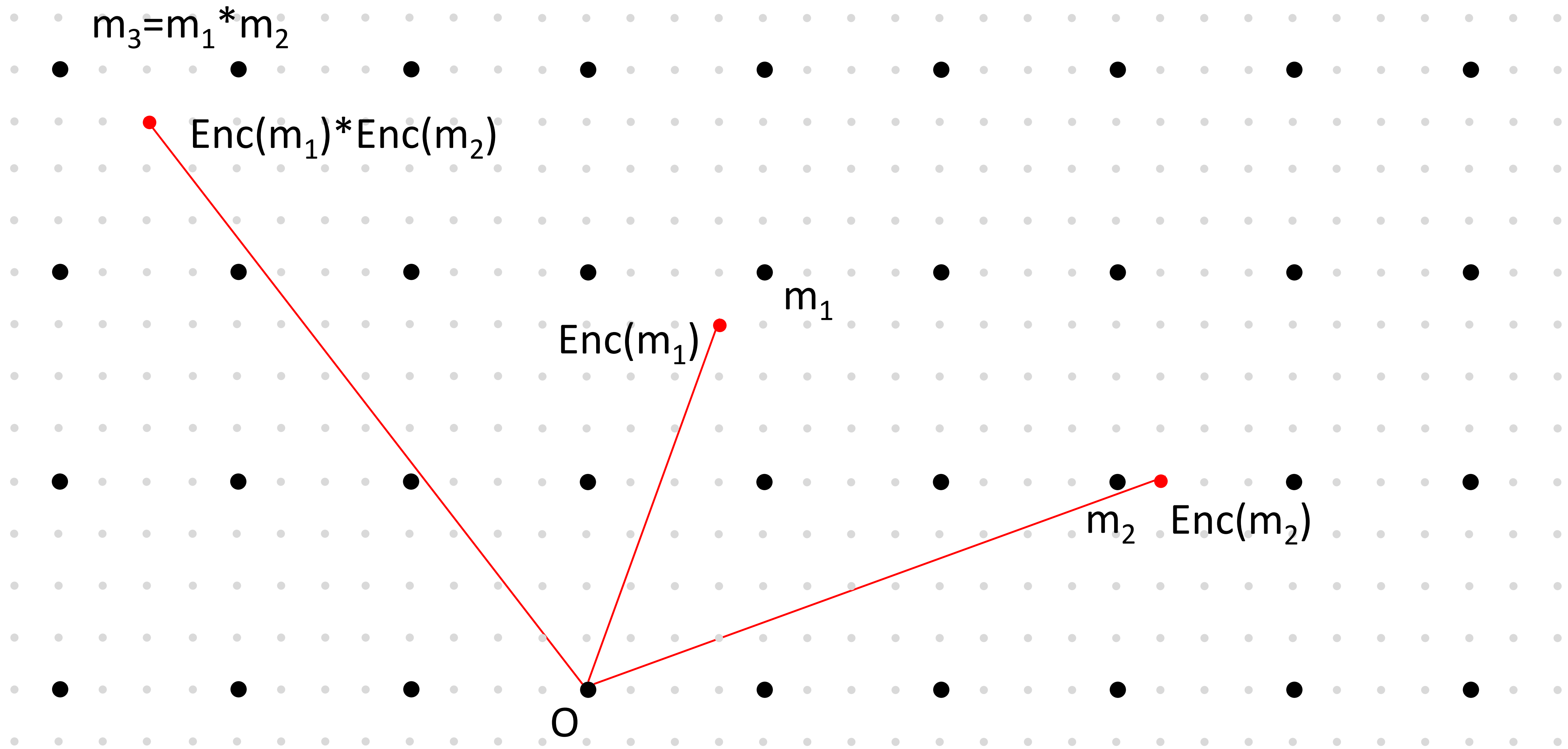

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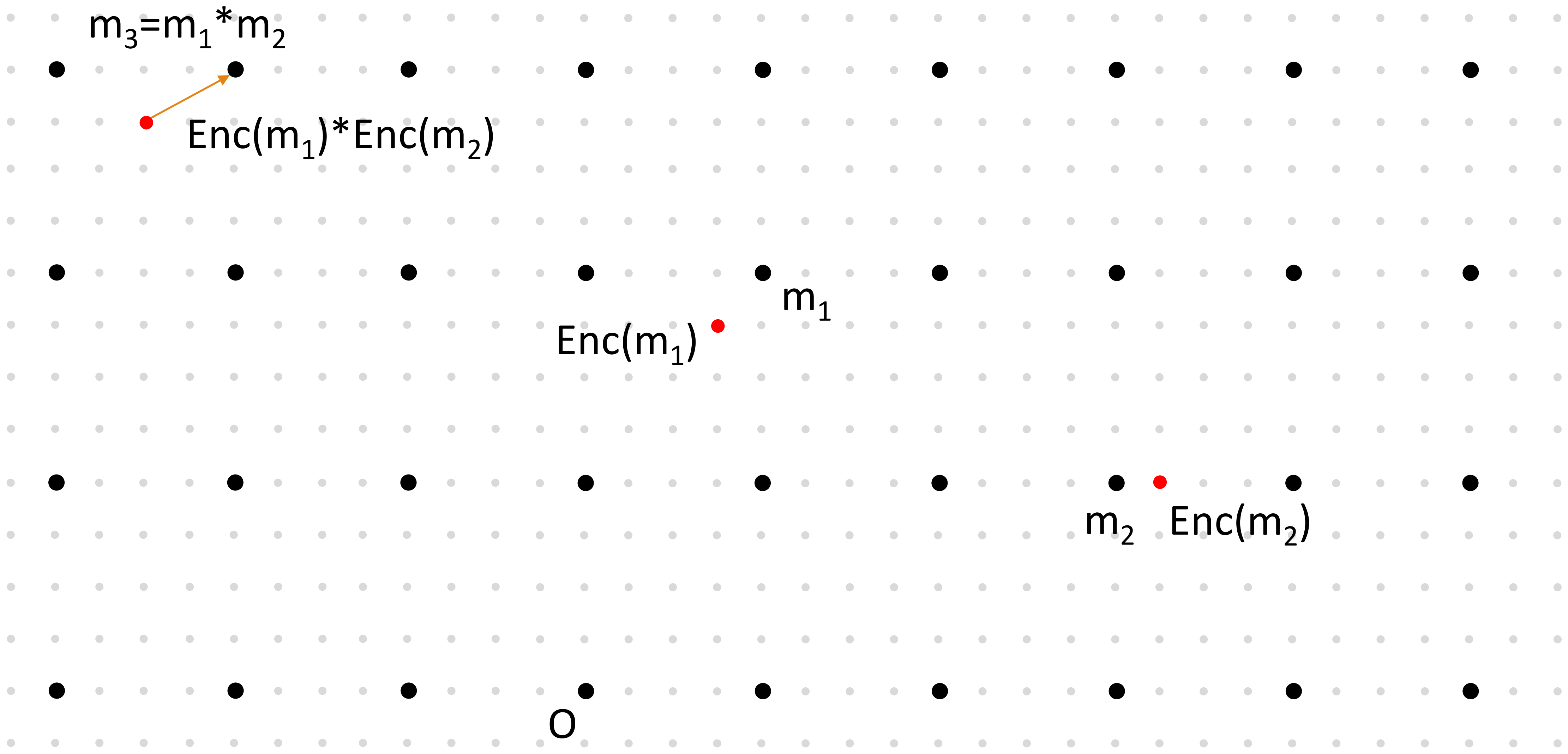

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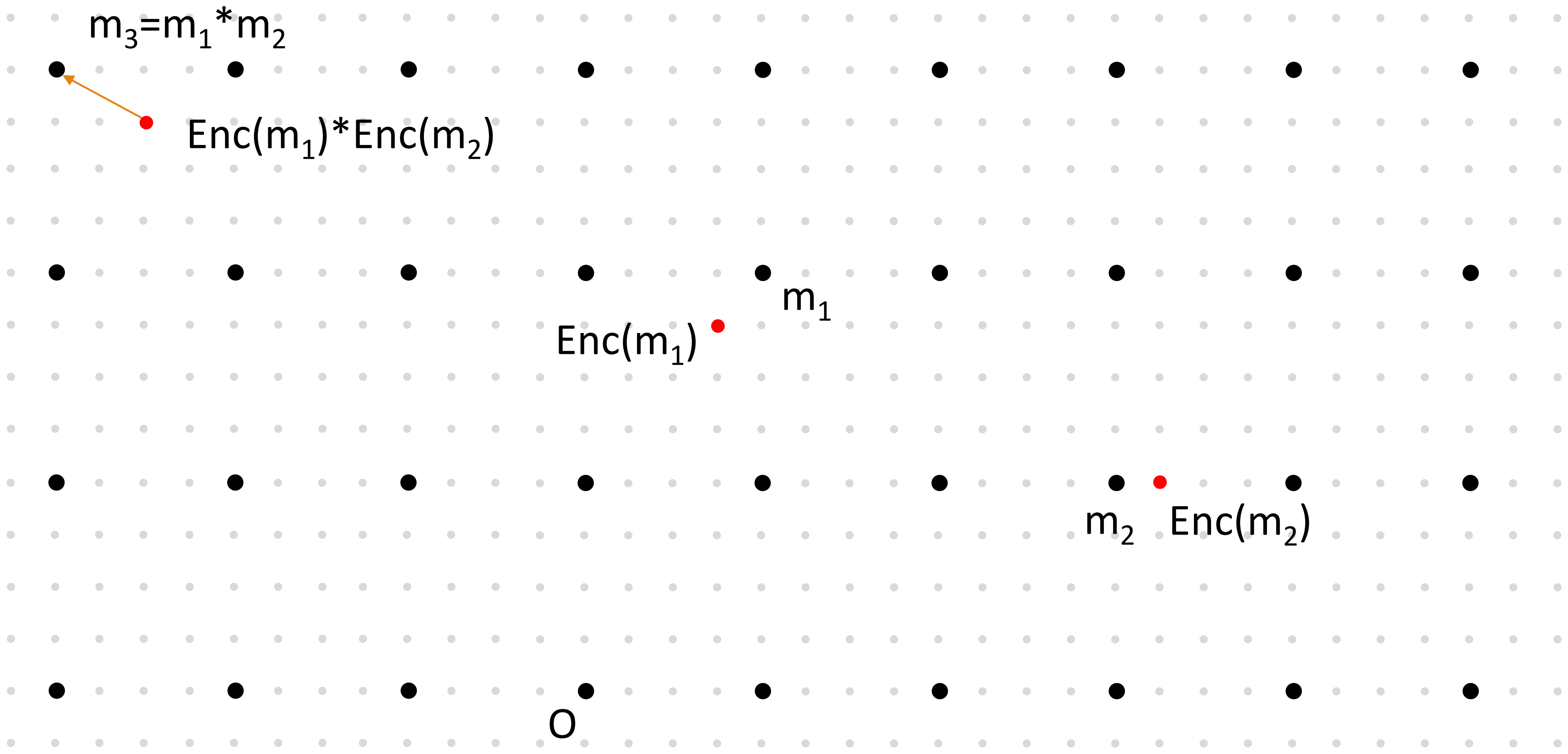

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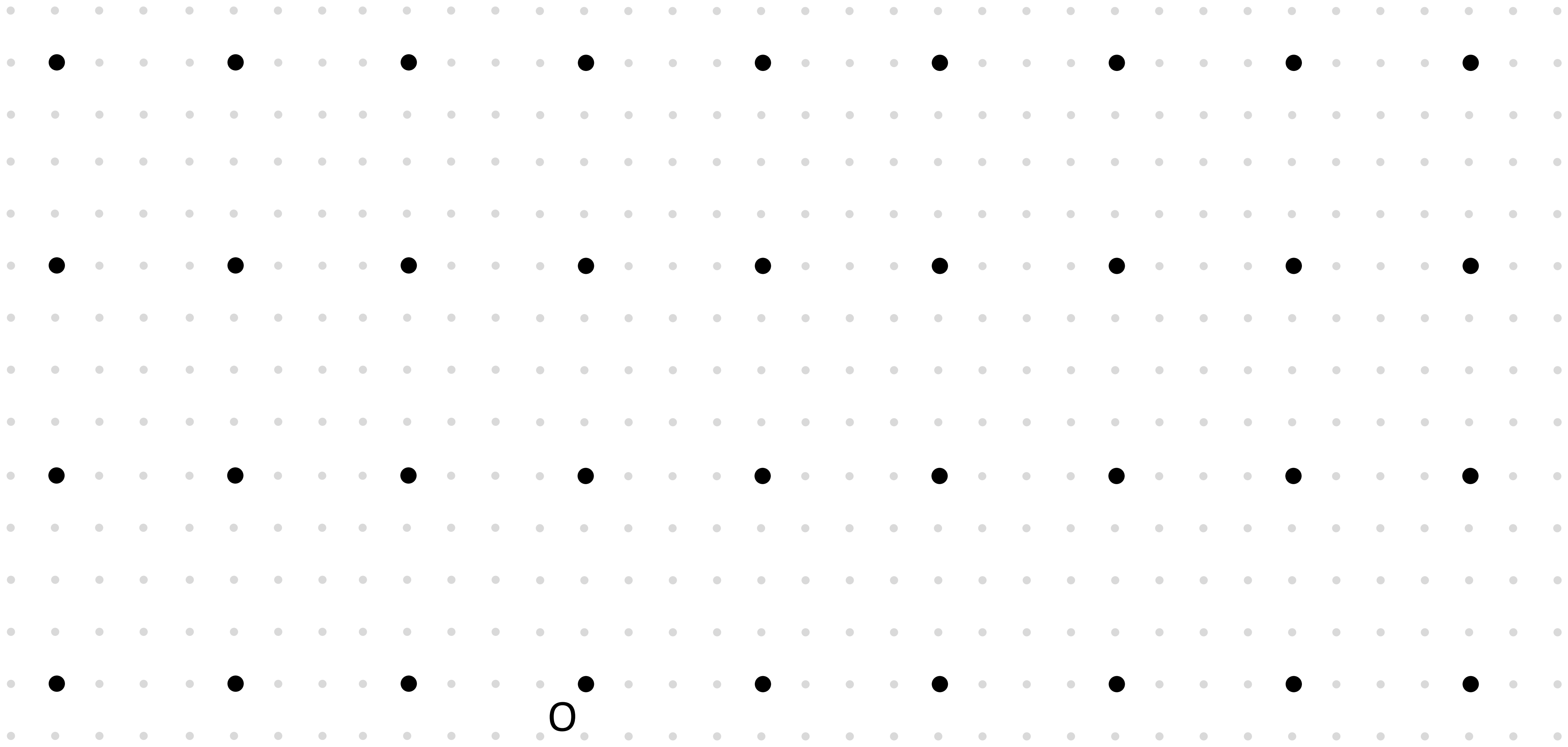

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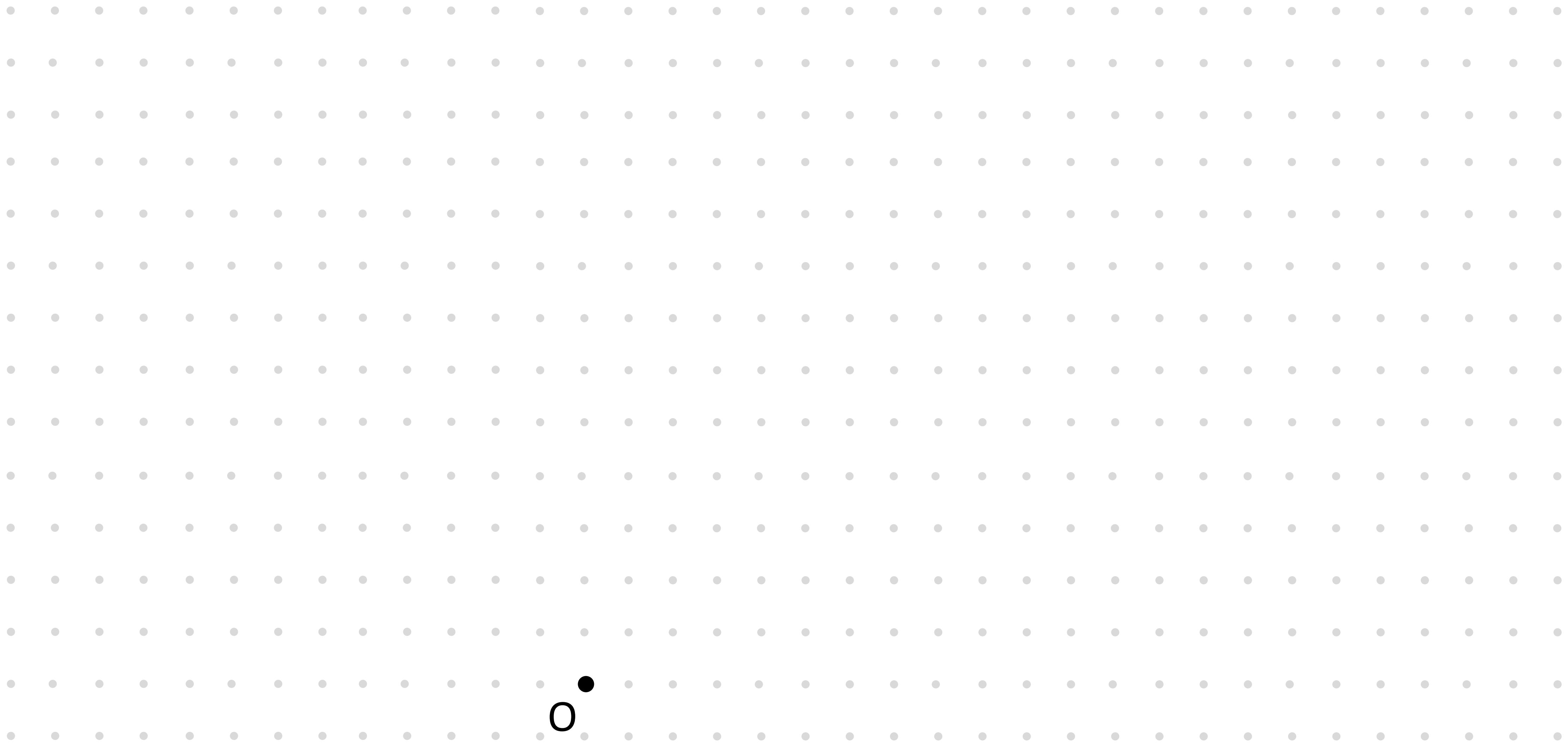
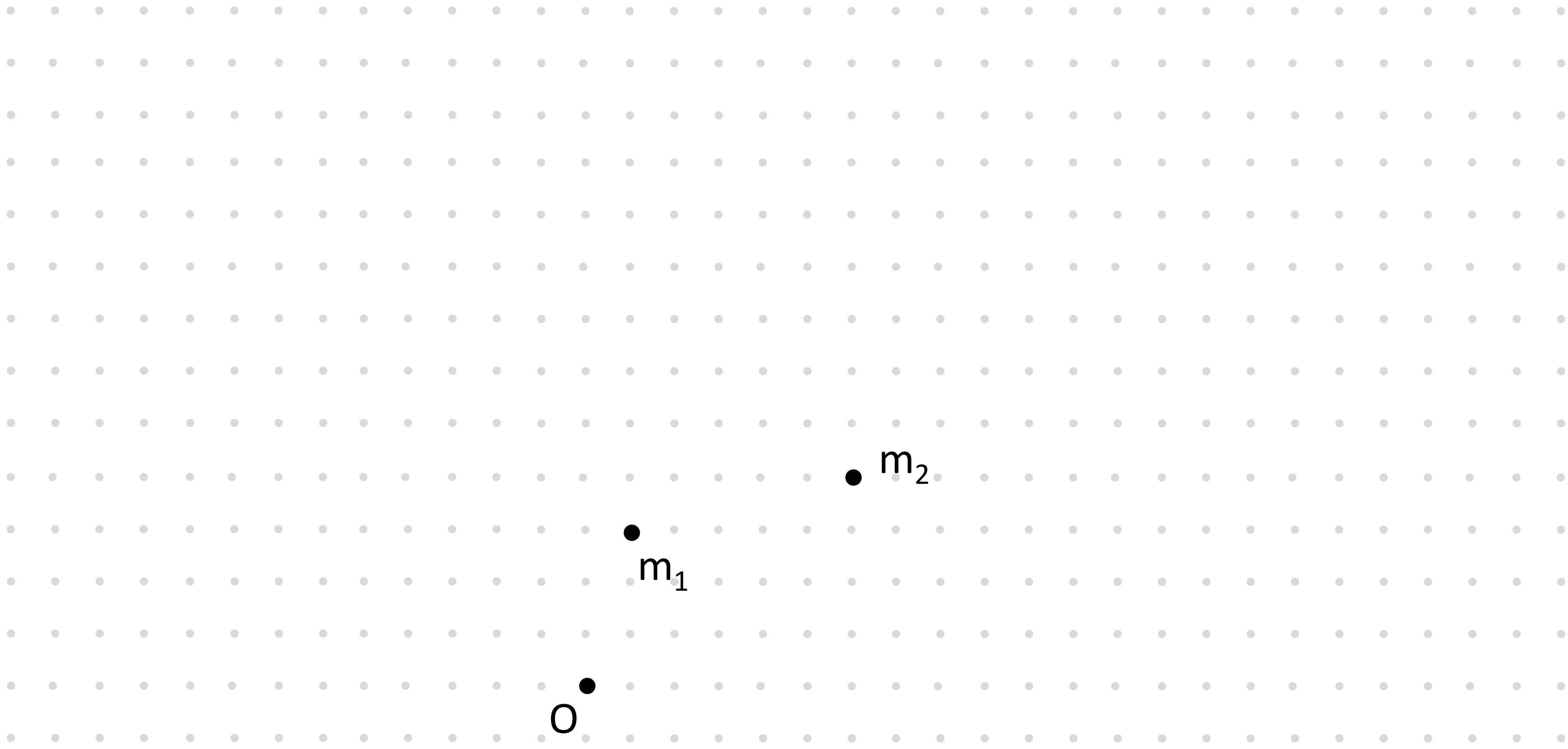
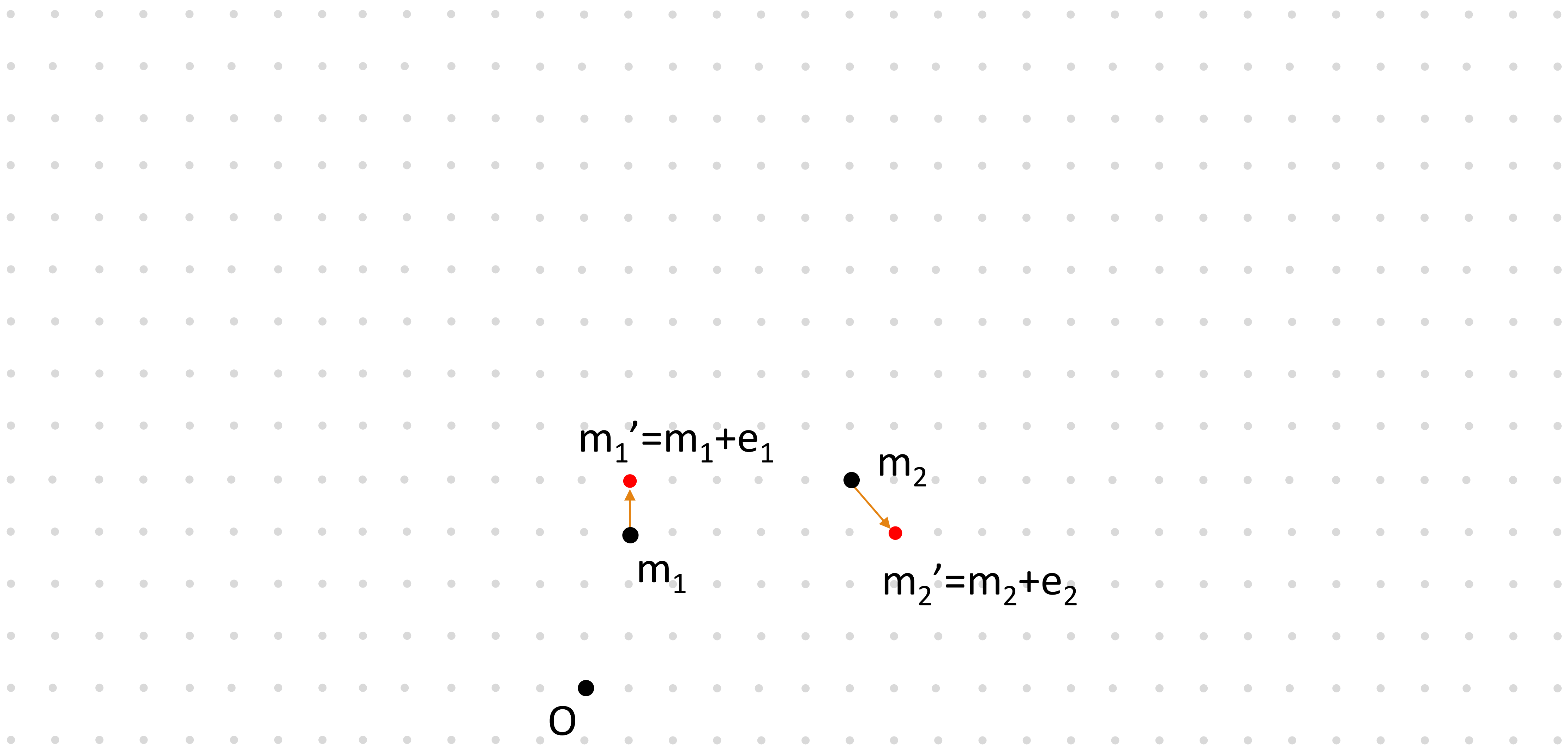


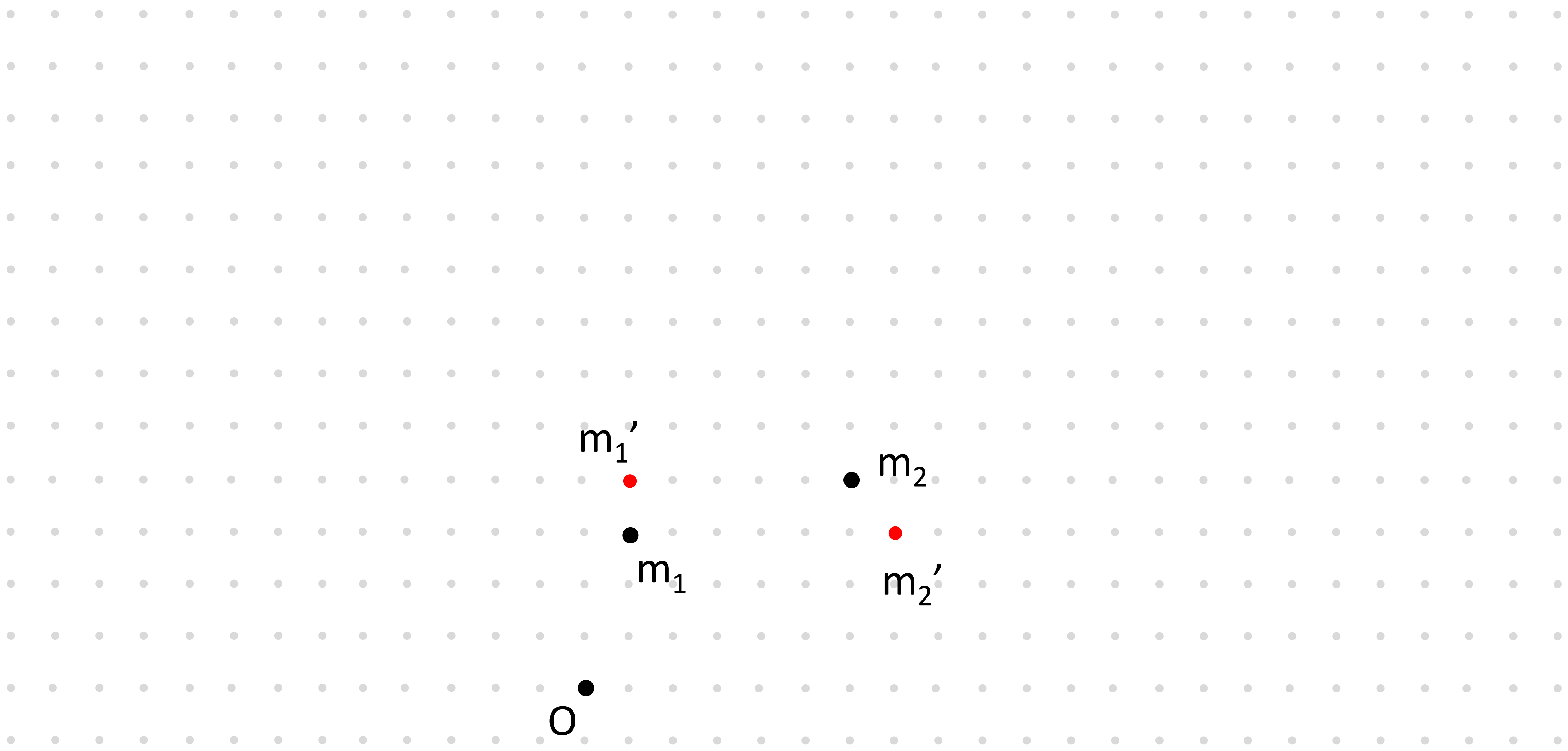
Approximate Homomorphic Encryption



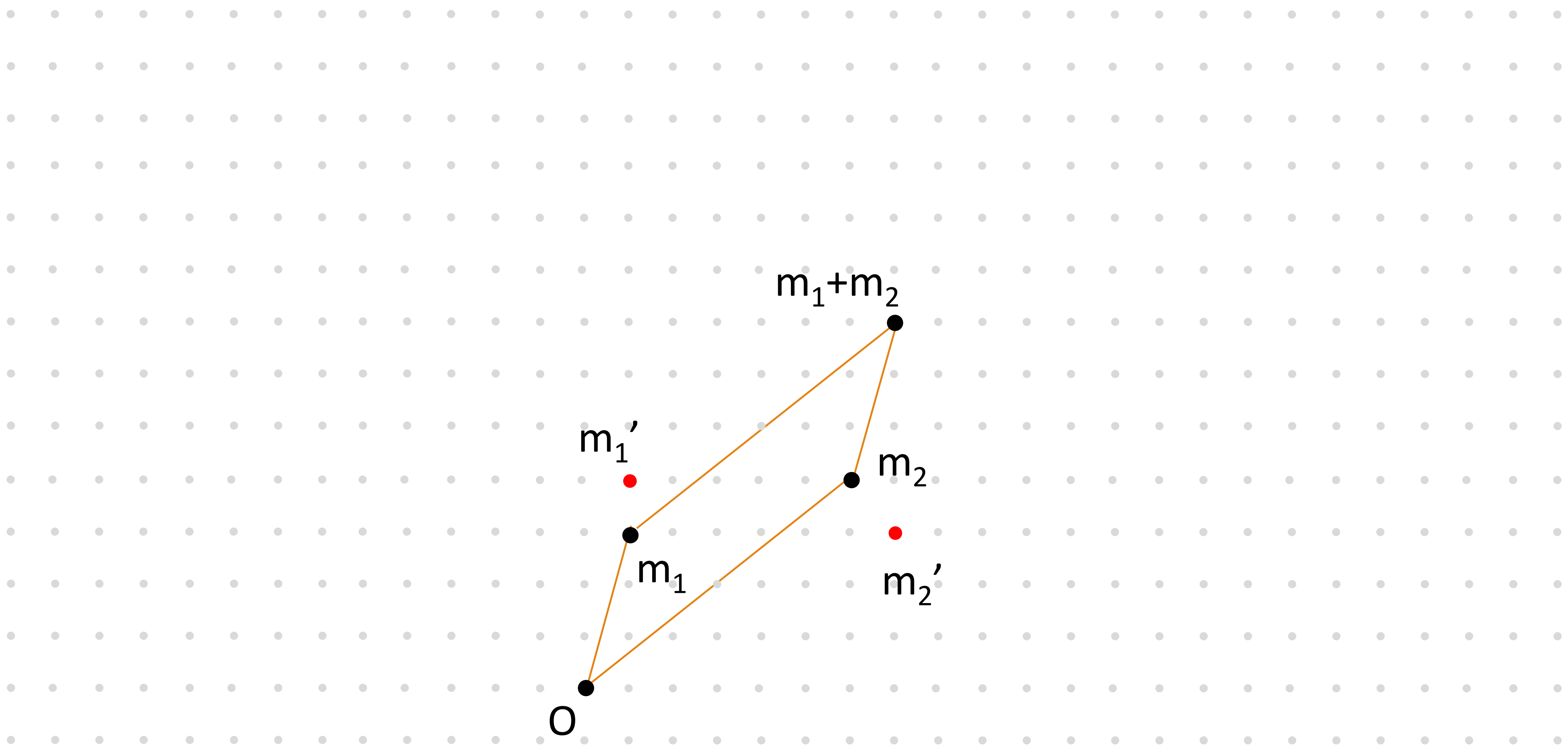
Approximate Homomorphic Encryption (Encryption)



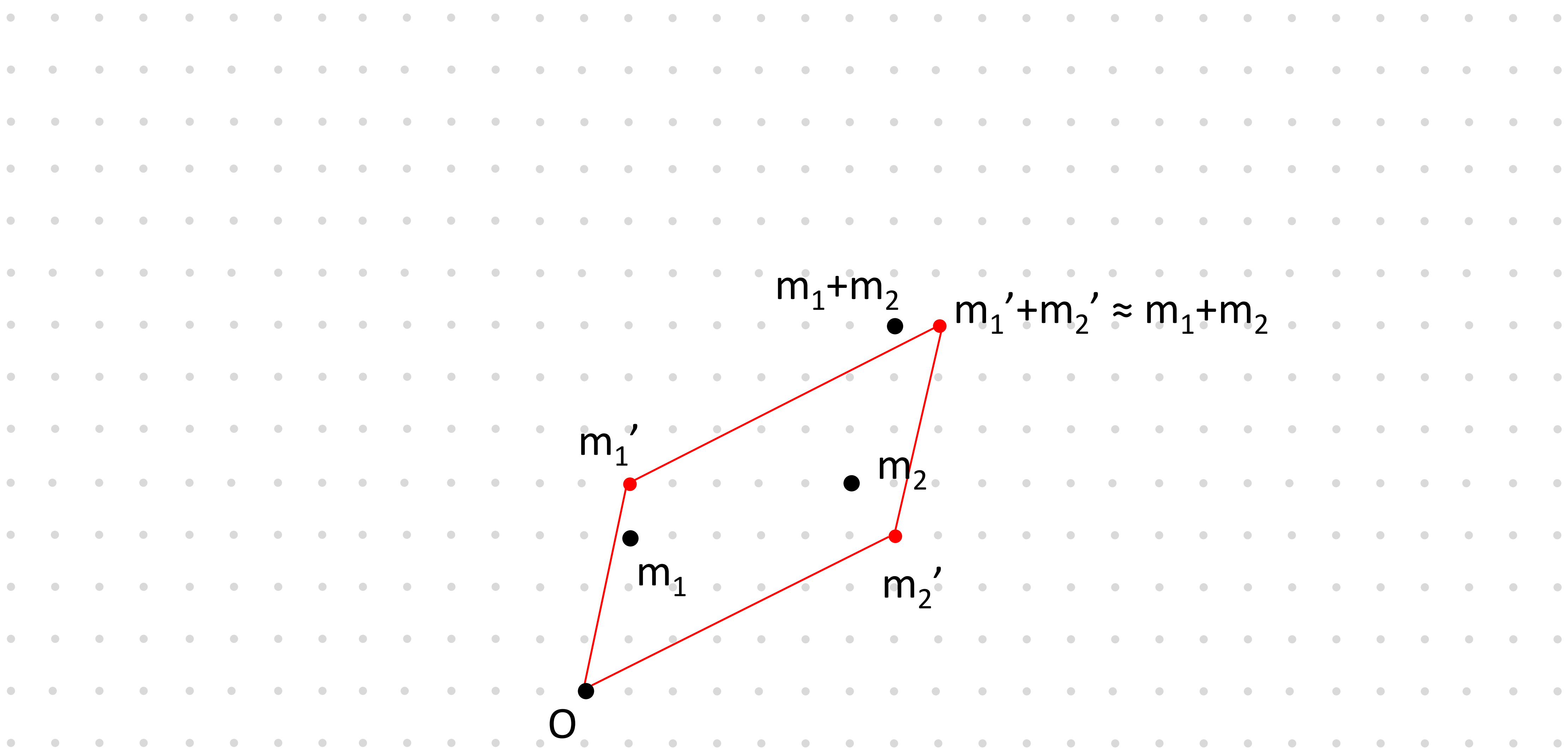
Approximate Homomorphic Encryption (Operations)



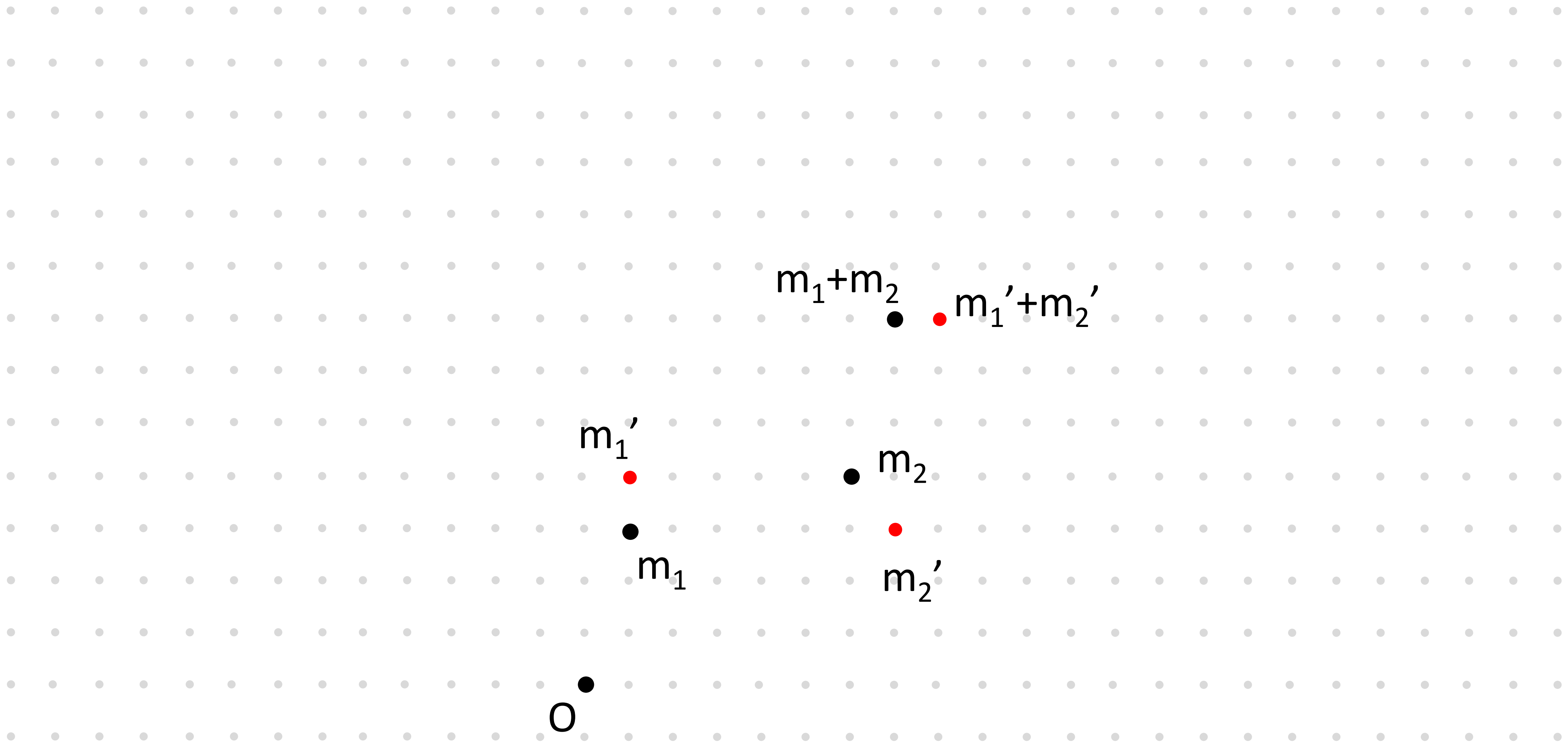
Approximate Homomorphic Encryption (Operations)



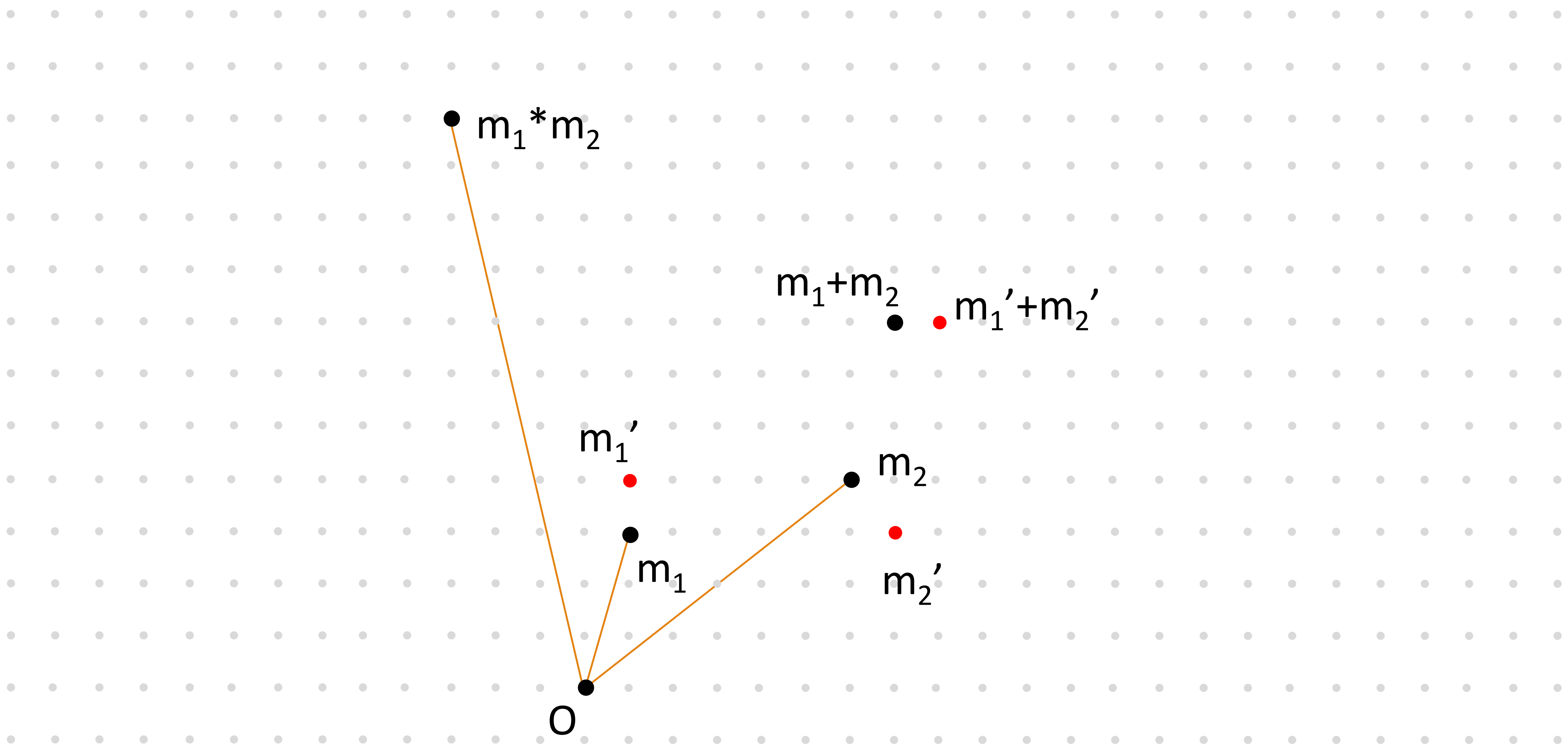
Approximate Homomorphic Encryption (Operations)



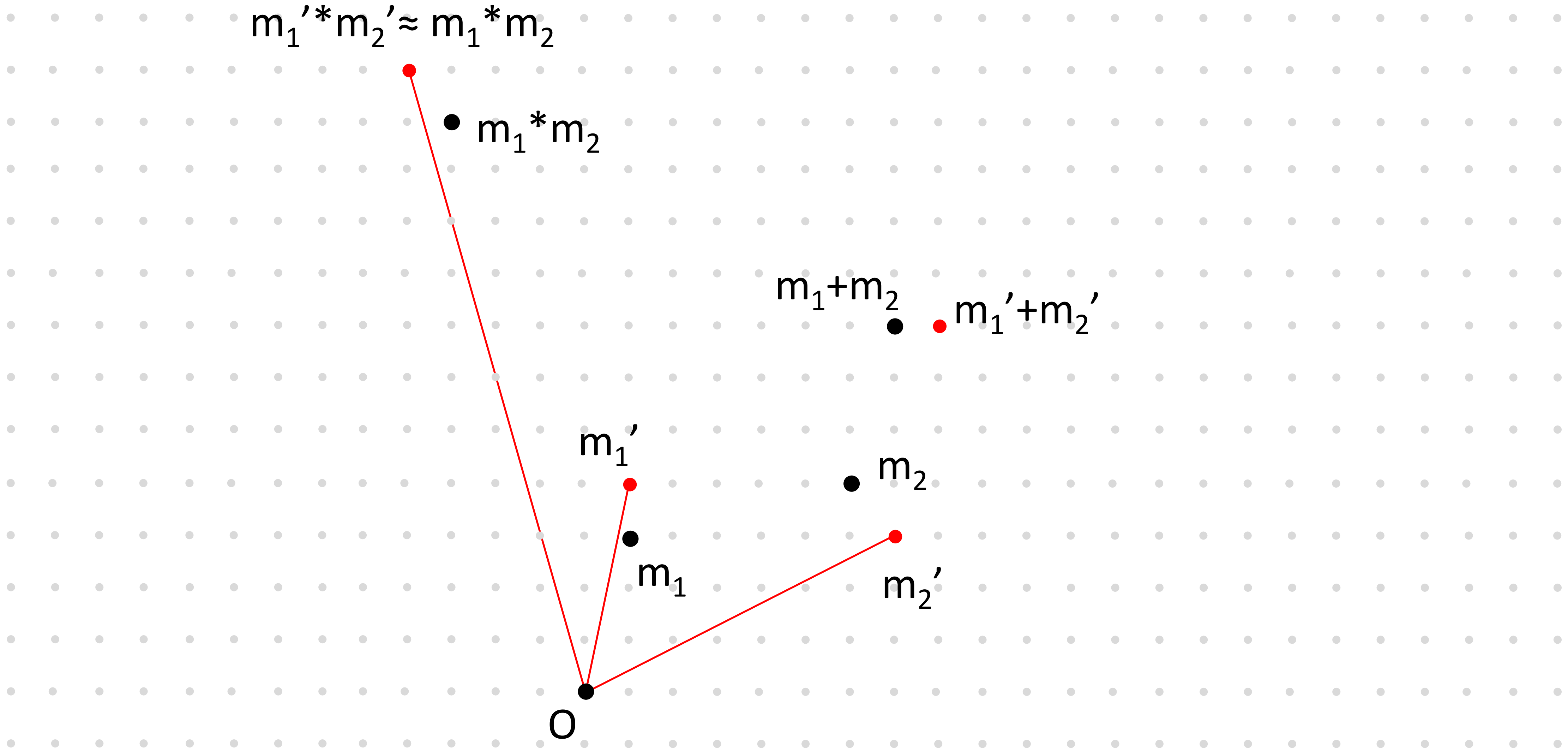
Approximate Homomorphic Encryption (Operations)



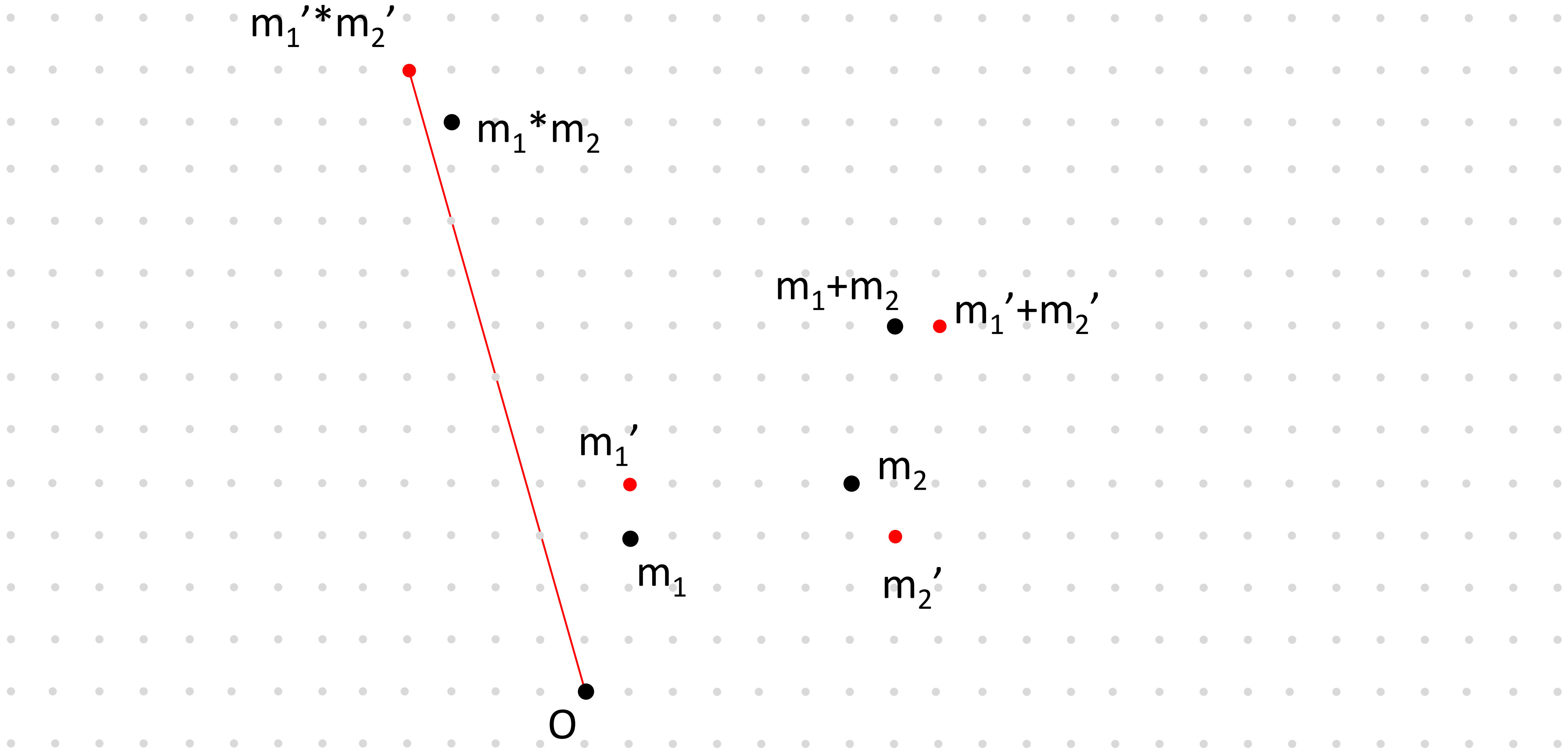
Approximate Homomorphic Encryption (Operations)



Approximate Homomorphic Encryption (Operations)



Approximate Homomorphic Encryption (Operations)



Approximate Homomorphic Encryption (Operations)